

# Analyzing Positioning Strategies in Sponsored Search Auctions Under CTR-Based Quality Scoring

Yong Yuan, Daniel Zeng, Huimin Zhao, and Linjing Li

**Abstract**—Quality score (QS) plays a critical role in sponsored search advertising (SSA) auctions, and in practice is closely correlated to the historical click-through rate (CTR) of an advertisement. The CTR-QS correlation may impose great influence on advertisers' positioning strategies of selecting the targeting slots in the sponsored list. In the literature, however, QS is implicitly assumed to be an independent variable and exogenously assigned by Web search engines, so that little theoretical or managerial insights can be offered to help understand the positioning dynamics in SSA auctions with CTR-QS correlation. We strive to bridge this research gap in this paper. Based on a discrete time-dependent optimal control model, which explicitly captures the relationship between the historical CTR and QS, we determine the optimal strategy for revenue-maximizing advertisers' QS-based positioning decisions through a policy-iteration-based numerical approximation method. We also investigate two practically-used heuristic strategies, namely the greedy and farsighted positioning strategies, aiming to examine and help understand advertisers' real-world positioning dynamics. Our analysis indicates that both the optimal and greedy positioning strategies lead advertisers to monotonically increase or decrease their targeting slots over time, which may cause a polarization trend emerging in SSA markets. Meanwhile, the farsighted positioning strategy can accelerate the polarization. Our simulations show that both the greedy and farsighted strategies have good revenue performance. Our findings indicate that advertisers should monotonically adjust their targeting positions to maximize their revenue in CTR-QS correlated SSA auctions. Our findings also highlight the need for Web search engine companies to set a lowered weight for historical CTRs or use position-normalized CTRs in their QS measurements, so as to suppress the polarization trend.

**Index Terms**—Click-through rate (CTR), optimal control, polarization, quality score (QS), sponsored search.

## I. INTRODUCTION

**S**PONSORED search advertising (SSA), also known as pay-per-click advertising or keyword advertising, is a prevalent business model in the Internet commerce industry, and a substantial revenue source for major Web search engine companies, such as Google and Yahoo! The basic economic instrument behind most SSA platforms is keyword auction (also called position auction), in which Web advertisers bid for their query-specific advertisements to appear above or alongside the organic search results. In order to ensure that superior advertisements are shown in more prominent positions so as to improve the user experience, most search engine companies are now using quality scores (QSs) to assess the quality of sponsored advertisements, and ranking them in descending order of advertisers' QS-weighted bids in SSA auctions.<sup>1</sup> With SSA platforms evolving in recent years, QS has been able to play a central role in the SSA ecosystem as a measurement to filter inferior advertisements, rank qualified advertisements and determine advertisers' cost-per-clicks (CPCs). Typically, advertisements with higher QSs will lead to more impressions, better positions, and lower CPCs.

As a user-oriented, data-intensive industry, Web search engine companies rely heavily on historical click-data in designing and improving their QS measurements. Although the exact formula of QS is kept a business secret and continually adjusted, major SSA platforms, including Google AdWords, Yahoo! Search Marketing, and Baidu Phoenix Nest, claim that the predominant factor in their QS measurements is the historical click-through rates (CTRs) of sponsored advertisements, and those advertisements receiving more clicks tend to get higher QSs.<sup>2</sup> This CTR-QS correlation has been intensively discussed and empirically observed by SSA practitioners.<sup>3</sup> As anecdotal evidence, empirical research on a

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<sup>1</sup>We use the term "quality score" to refer to Google's quality score and Yahoo!'s quality index, or any measurement of the same nature used by other search engines.

<sup>2</sup><https://support.google.com/adwords/certification/bin/static.py?hl=en&topic=23688&guide=23686&page=guide.cs&answer=152234>; [http://is.baidu.com/bidding\\_price.html](http://is.baidu.com/bidding_price.html); and [http://help.yahoo.com/l/uk/yahoo/ysm/sps/start/overview\\_qualityindex.html](http://help.yahoo.com/l/uk/yahoo/ysm/sps/start/overview_qualityindex.html)

<sup>3</sup><http://www.searchenginejournal.com/google-quality-score-click-through-rate/9936/>

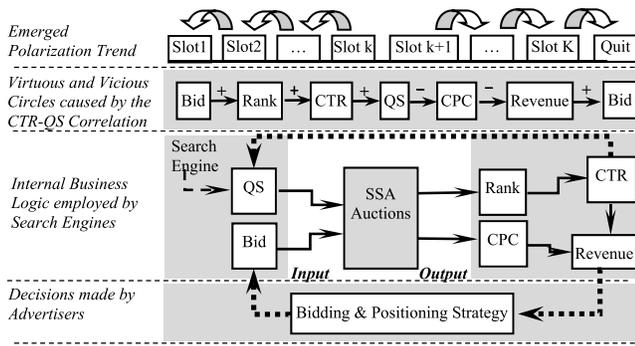


Fig. 1. CTR-QS correlation and the emerged polarization trend.<sup>6</sup>

Google account with more than 500 thousands keywords indicates that the QS can be very well explained by historical CTR ( $R^2 = 72\%$ ).<sup>4</sup> Meanwhile, our linear regression analysis on the data released by an SSA agency shows that QS and CTR are closely correlated with  $R^2 = 82\%$  in all eight advertisement slots.<sup>5</sup>

This correlation will incur the CTR-cumulative effect on the QS measurements in SSA auctions. As it is officially publicized by Yahoo! Search Marketing, “as impressions and clicks accumulate, CTR becomes a stronger factor in calculating the QS.”<sup>7</sup> This CTR-cumulative effect may impose great influence on advertisers’ behavioral dynamics in SSA auctions, but has not been studied and documented in the existing research. In [1]–[3], the QSs and bids of sponsored advertisements are exogenously given as inputs and manipulated in SSA auctions to generate the ranks and CPCs, which in turn help determine the CTRs and advertisers’ revenue, as shown in Fig. 1. In this modeling logic, QS is implicitly assumed to be an independent variable that is exogenously assigned by search engines, and remains unchanged in repeated SSA auctions. As a result, the SSA auction is a static system from the perspective of advertisers, with their revenue controlled only by their bidding and positioning strategies. Under this key assumption of QS independence, it is widely believed in the literature (especially the research works from a game theoretic viewpoint) that advertisers will finally reach an efficient Nash equilibrium state (e.g., the locally envy-free equilibrium [4]), with high-valued advertisers in prominent slots and low-valued advertisers in inferior slots, as illustrated on the left side of Fig. 2. No advertisers would have incentive to change their bid and slot in the equilibrium state.

However, as aforementioned, the QS of an advertisement in fact is largely determined by its historical CTR and keeps changing due to cumulating clicks, although this cumulative effect decreases over time. As such, from

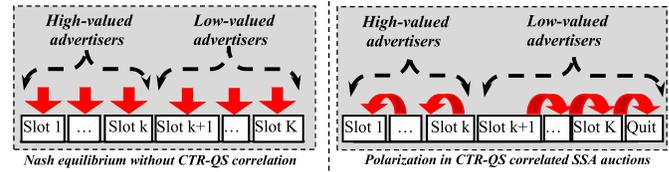


Fig. 2. Positioning dynamics in SSA auctions with or without CTR-QS correlation.

a modeling perspective, QS is not an exogenous constant, but an endogenous variable updated by the historical CTR in repeated SSA auctions. Therefore, the assumption of QS independence used in the literature is impractical. This motivates us to relax the assumption, and revisit advertisers’ positioning dynamics in CTR-QS correlated SSA auctions.

As can be seen from Fig. 1, if historical CTR is used as a feedback for updating the QS, the SSA auction becomes an inherently dynamic system with two closed-loops (illustrated by dotted lines). Due to this CTR-QS correlation, virtuous and vicious circles may emerge in advertisers’ positioning behavior. For instance, advertisers with a high per-click value will be motivated to raise their bid to get a higher rank and CTR [5].<sup>8</sup> Due to the CTR-cumulative effect on the QS, the increased CTR will help raise the QS of their advertisement and in turn decrease the CPC. With a lowered CPC and an increased CTR, the revenue increases, and thus an advertiser can afford to bid more aggressively, resulting in the virtuous circle. As a result, the advertiser can keep moving upward in the sponsored list with an improving QS and revenue, until finally stabilizing in the slot that meets some specific conditions (determined by Theorem 3 in this paper). Analogously, an advertiser with a lower per-click value has to decrease the bid and get an inferior slot with a lowered CTR. The lowered CTR will lead to a decrease in the QS, and in turn an increase in the CPC of the advertisement. The increased CPC will reduce the revenue of the advertiser, who thus has to further decrease the bid and move downward in the sponsored list until stabilizing in a specific slot, resulting in the vicious cycle. When the CPC in the bottom slot increases above the advertiser’s per-click value, he or she will not afford to compete and thus quit the SSA auctions. To sum up, due to the CTR-QS correlation, advertisers will be motivated to monotonically increase or decrease the targeting slots through adjusting their bids, and thus may cause divergent positioning dynamics of virtuous and vicious cycles and in turn a polarization trend in SSA markets, as can be illustrated on the right side of Fig. 2.

From a research perspective, there is a critical need to characterize and help understand advertisers’ positioning dynamics caused by the CTR-QS correlation in SSA auctions, targeting at identifying the optimal positioning strategies for advertisers to better exploit the CTR-cumulative effect on their QSs in

<sup>4</sup><http://searchengineland.com/how-important-is-click-through-rate-in-googles-quality-score-formula-27296>

<sup>5</sup><http://www.epiphanysearch.co.uk/blog/decoding-the-quality-score-2/>

<sup>6</sup>Here we use “+” and “-” to denote the positive and negative impacts between variables, respectively. For instance, the virtuous circle can be depicted as “ceteris paribus, an increased bid leads to a higher rank and CTR, and in turn an increased QS and lowered CPC, and then higher revenue and further increased bid.” The vicious circle can be depicted analogously.

<sup>7</sup>[http://help.yahoo.com/l/uk/yahoo/ysm/sps/start/overview\\_qualityindex.html](http://help.yahoo.com/l/uk/yahoo/ysm/sps/start/overview_qualityindex.html)

<sup>8</sup>In this paper, we use the terms “position,” “slot,” and “rank” interchangeably.

pursuit of maximized revenue, and also offering useful policy suggestions for Web search engine companies in improving their QS measurements. However, to the best of our knowledge, research works in this area are still nonexistent, and our research represents a first attempt to study this problem.

### A. Literature Review

We here briefly survey two branches of related works concerning advertisers' bidding and positioning strategies in SSA auctions.

Online auction has long been a hot research topic [6]–[10]. With respect to advertisers' bidding behavior in SSA auctions, researchers have conducted intensive analyses on the landscape and revenue properties of the pure-strategy Nash equilibrium continuum of the SSA auction games and its refinements [11]–[13], including symmetric Nash equilibrium [14], [15] or locally envy-free equilibrium equivalently [4]. Different kinds of strategic behavior, such as greedy bidding [16], [17], vindictive bidding [18], [19], and cyclical bidding [20], [21], have been observed, analyzed, and documented. Most of these research efforts assume that search engines use the "rank-by-bid" SSA auctions without considering the QSs in ranking advertisements [22]. Lahaie and Pennock [1] and Yoon [2] further analyzed the equilibrium bidding behavior and the revenue properties in "rank-by-revenue" SSA auctions, where advertisements are ranked by QS-weighted bids. As aforementioned, these research efforts come to the similar conclusion that advertisers will reach a specific equilibrium state with no advertiser willing to change position.

Concerning advertisers' positioning strategies, Aggarwal *et al.* [23] argued that the top slots might not be the most profitable position for Web advertisers. Jerath *et al.* [24] further empirically observed a counter-intuitive "position paradox" that superior advertisers tends to bid for lower slots while inferior advertisers will compete for top slots, which has been supported by simulation research [25].

The above research efforts have offered an in-depth characterization about advertisers' behavior dynamics in SSA auctions. A good survey on these lines of research can be found in [26]. However, these efforts either do not take the QS of an advertisement into consideration, or implicitly assume that the QS is exogenously assigned by search engines and is independent to the historical CTR. As such, little theoretical and managerial insights are offered to help understand advertisers' behavior dynamics in CTR-QS correlated SSA auctions. This paper is aimed at filling in this important research gap.

### B. Contributions and Implications

In this paper, we explicitly take into consideration the CTR-QS correlation in SSA auctions, and formulate advertiser's QS-based positioning decisions as a discrete time-dependent optimal control problem. In our model, advertisers seek to maximize their revenue by optimizing the positioning strategy according to the immediate QS in each single

auction session of the repeated SSA auctions. Using policy iteration, we numerically determine the optimal positioning strategy for advertisers' rank selection in SSA auctions.<sup>9</sup> We also analyze the behavior dynamics of advertisers using two widely-used and heuristic positioning strategies, namely, the greedy strategy that maximizes advertisers' immediate revenue in each stage and the farsighted strategy that maximizes the discounted cumulative revenue in one specific slot in all stages. Using simulations, we numerically compare the revenue performance of these three positioning strategies (i.e., optimal, greedy, and farsighted).

Our key finding is that, due to the CTR-cumulative effect on the QS, advertisers using either of the optimal or the greedy positioning strategies will monotonically increase (for advertisers with high per-click values) or decrease (for advertisers with low per-click values) their targeting slots. As such, our finding indicates that a polarization trend may emerge in the SSA markets. Furthermore, although the farsighted strategy does not cause the divergent dynamics of slot adjustments due to its stationarity, it can indirectly help accelerate this polarization trend. Note that it has been empirically witnessed that the average CPC in SSA markets has significantly decreased and increased for advertisers on the higher and lower slots, respectively, during 2009–2011.<sup>10</sup> Our findings on the polarization trend of advertisers' QSs and ranks offer a potential explanation for the underlying dynamics behind this phenomenon.

Our findings have important implications for both advertisers and Web search engine companies. For advertisers, this paper indicates that in order to maximize their revenue in SSA auctions with CTR-QS correlation, advertisers should monotonically adjust their targeting slots, in terms of the per-click values and the QSs of their advertisements. We also present three strategies to help revenue-maximizing, greedy, and farsighted advertisers formulate their positioning decisions, respectively. Our experiments indicate that both the greedy and farsighted strategies have good revenue performance, and in practice can serve as effective alternatives to the optimal strategy. For Web search engine companies, our research indicates that the CTR-QS correlation may result in a polarization trend in advertisers' positioning dynamics. Our finding also highlights the need for Web search engine companies to set a lowered weight for historical CTRs or use position-normalized CTRs in their QS measurements.

The remainder of this paper is organized as follows. Section II presents an SSA auction scenario and some assumptions, and a formal model for the decision-making problem faced by advertisers. In Section III, we numerically determine advertisers' optimal positioning strategy using an

<sup>9</sup>Note that with an optimal targeting rank, an advertiser can easily find the optimal bid or bid interval for that slot by experimenting on the SSA markets through shifting their bids and submitting dummy search requests. Also, there are lots of online services and software tools aiming at adjusting advertisers' bids in a real-time fashion to maintain a specific rank. As such, we focus our analysis on advertisers' positioning strategies, instead of bidding strategies. It is worth noting that the optimal positioning strategies discussed in this paper are also revenue-maximizing strategies.

<sup>10</sup>[http://www.adgooroo.com/has\\_google\\_changed\\_their\\_cpc\\_formula.php](http://www.adgooroo.com/has_google_changed_their_cpc_formula.php)

algorithm based on policy iteration. In Sections IV and V, we discuss the behavior dynamics of advertisers using the greedy and farsighted strategies, respectively. In Section VI, we compare the revenue performance of the investigated strategies, discuss advertisers' positioning strategies in SSA auctions with incomplete information settings, and present some potential solutions for dealing with the polarization trend in SSA markets. Section VII concludes our efforts and discusses some future research directions.

## II. SSA AUCTION SCENARIO AND THE MODEL

We begin our analysis by describing a typical SSA auction scenario, presenting some assumptions, and then developing the formal model for analyzing an advertiser's positioning strategy.

### A. SSA Auction Scenario

We now introduce an SSA model shared by [12] and [14]. We consider an SSA auction scenario with  $N$  advertisers competing for  $K$  slots on a given keyword, where  $N > K \geq 2$ . The CTR of the  $k$ th highest slot is denoted by  $x_k$ . We assume that higher-placed slots have higher probabilities of being clicked [5], and thus  $x_1 > x_2 > \dots > x_K > 0$ .<sup>11</sup> For notational simplicity, we add  $N - K$  dummy slots with zero CTRs, so that  $x_k = 0$  for  $k > K$ . The per-click value and the bid of the advertiser in the  $k$ th highest slot are denoted by  $v_k$  and  $b_k$ , respectively, and the QS of that person's advertisement is denoted by  $q_k$ .

Once a query arrives, the search engine starts a generalized second price (GSP) auction and allocates advertisers to advertisement slots in decreasing order by their QS-weighted bids, which are called ad-ranks by Google. We denote the ad-rank of the  $k$ th highest slot by  $h(k) = q_k b_k$ , and thus  $h(1) \geq h(2) \geq \dots \geq h(N)$ . The payment rule of a GSP auction follows the "second-price" scheme: advertisers in slot  $k$  shall pay  $h(k+1)/q_k$  each time their advertisement is clicked, with the revenue realized as  $u_k = (v_k - h(k+1)/q_k)x_k$ .

### B. Assumptions

We make the following simplifying assumptions.

*Assumption 1:* Advertisers have complete information about the ad-rank and CTR of each slot.

This is reasonable as it is relatively easy for advertisers to experiment on the market by shifting their bids and submitting dummy search requests to test the rank changes, so as to come up with an estimation of the ad-rank in each slot [14], [27]. Besides, various kinds of services and analytical reports are online available to help estimate the CTRs of all slots.<sup>12</sup>

*Assumption 2:* The QS of an advertisement is determined by its historical CTR.

<sup>11</sup>In the literature, CTR can be separated into an advertiser effect  $\alpha$  and a positional effect  $\beta$ , and  $\beta_1 > \beta_2 > \dots > \beta_K > 0$ . Put differently, an advertiser  $i \in [1, N]$  on slot  $k \in [1, K]$  will get a CTR of  $x_{ik} = \alpha_i \beta_k$ . Since we focus on the positioning decision of a single advertiser,  $\alpha_i$  will be constant, so that we use  $x_k$  to replace  $x_{ik}$  and can reasonably assume  $x_1 > x_2 > \dots > x_K > 0$ .

<sup>12</sup><http://www.smartinsights.com/search-engine-optimisation-seo/seo-analytics/comparison-of-google-clickthrough-rates-by-position/>

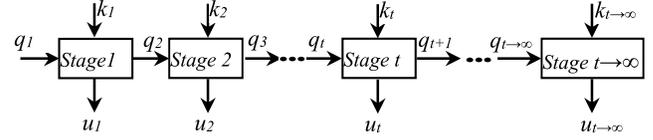


Fig. 3. Multistage decision optimization process.

Before 2005, Google AdWords ranked advertisements purely by their bids weighted by CTRs.<sup>13</sup> Although other factors, such as landing page quality and keyword-query relevance, are taken into consideration later on, historical CTR has always been the predominant factor in the QS measurements in Google AdWords and most other SSA platforms.<sup>14</sup> Since this paper focuses on the CTR-QS correlation and its impacts on advertisers' positioning behavior, we thus ignore other factors and assume that the QS of an advertisement is determined by (equals) its historical CTR.

*Assumption 3:* The volume of search requests (impressions) is stable in each stage.

We do not consider the fluctuations of search volume caused by seasons, holidays or sudden events, so that the volume of advertisement impressions can be considered stable over time.

### C. Model

We focus on the decision-making problem faced by advertisers in repeated SSA auctions with CTR-QS correlation. In practice, advertisers usually check their QS regularly (e.g., daily). After observing an immediate QS in each stage, the advertiser has to make a decision about the optimal targeting slot and adjust the bid accordingly. Since the QS is determined by the historical CTR, the CTR of the selected slot in the current stage will affect the QS in the next stage. Therefore, the advertiser faces a multistage decision optimization problem, with the objective of maximizing the discounted cumulative revenue in the long run. The decision process can be depicted in Fig. 3, in which the notations are defined in the model below.

We develop the following discrete time-dependent optimal control model to analyze an advertiser's positioning strategy. The model is specified by a seven-tuple  $(\mathcal{Q}, \mathcal{K}, \mathcal{T}, \sigma, u, f, \delta)$ , as shown below.

- 1)  $\mathcal{Q} = (0, 1)$  is an infinite, continuous state space. A state  $q \in \mathcal{Q}$  denotes the QS of the advertisement in a specific stage.
- 2)  $\mathcal{K} = \{1, 2, \dots, K+1\}$  is a discrete action space. An action  $k \in \mathcal{K}$  denotes that the advertiser will bid for slot  $k$ , and  $k = K+1$  means that the advertiser chooses to quit the SSA auctions.
- 3)  $\mathcal{T} = \{1, 2, \dots\}$  is an infinite set of stages. Note that a stage does not need to refer to a fixed time interval, since advertisement impressions are assumed to be stable over time.

<sup>13</sup><http://www.ppchero.com/guides/history-of-adwords-quality-score-and-periodic-changes/>

<sup>14</sup>Also note that the empirical studies mentioned in Section I show that CTR is the most important factor in QS with  $R^2 = 72\%$  and  $R^2 = 82\%$ , respectively.

- 4)  $\sigma: \mathcal{Q} \times \mathcal{T} \rightarrow \mathcal{K}$  denotes a policy that determines an action  $k \in \mathcal{K}$  for each state  $q \in \mathcal{Q}$  in each stage  $t \in \mathcal{T}$ . Note that the term “policy” used throughout this paper also refers to “strategy.” In subsequent sections, these two words are interchangeably used.
- 5)  $u: \mathcal{Q} \times \mathcal{K} \rightarrow \mathcal{R}$  is a one-stage revenue function that specifies an immediate revenue for taking an action  $k \in \mathcal{K}$  in a state  $q \in \mathcal{Q}$ . Since we are studying the decision-making of one advertiser, we thus omit the subscripts in such advertiser-specific characteristics as the per-click value and QS. Also, following [14], we here focus on exogenous advertisers who need only to adjust their QS-weighted bid over  $h(k)$ , instead of  $h(k+1)$ , to obtain the slot  $k$  and thus pay  $h(k)/q$ . Hence, we have

$$u(q, k) = \left[ v - \frac{h(k)}{q} \right] x_k. \quad (1)$$

- 6)  $f: \mathcal{Q} \times \mathcal{K} \times \mathcal{T} \rightarrow \mathcal{Q}$  is the state transition function that specifies the following state in terms of the current stage, state, and action. Note that the QS equals the historical CTR of an advertisement. So if an advertiser with a QS of  $q_t$  targets the slot  $k_t$  in stage  $t$ , then we have

$$q_{t+1} = \frac{q_t * \sum_{i=0}^{t-1} c_i + x_{k_t} * c_t}{\sum_{i=0}^{t-1} c_i + c_t} \quad (2)$$

where  $c_t$  is the number of impressions that an advertisement gets in stage  $t$ , with  $c_0 > 0$  initially. According to Assumption 3, we have  $c_i = c_j$  for any arbitrary  $i, j \in 0, 1, \dots$ , and thus

$$q_{t+1} = f(q_t, k_t, t) = \frac{q_t t + x_{k_t}}{t+1}, \quad t = 1, 2, \dots \quad (3)$$

- 7)  $\delta \in (0, 1)$  is the discount factor.

To maximize the discounted cumulative revenue, an advertiser should find the optimal control law to determine a sequence of actions (i.e., targeting slots), each in a stage, after observing the initial QS. Formally, the optimal positioning strategy satisfies

$$\operatorname{argmax}_{k_t \in \mathcal{K}} \sum_{t=1}^{\infty} \delta^{t-1} u(q_t, k_t). \quad (4)$$

Subject to  $q_{t+1} = \frac{q_t t + x_{k_t}}{t+1}$ ,  $t = 1, 2, \dots$  and  $q_1$  is known.

### III. STRATEGY OPTIMIZATION BASED ON POLICY ITERATION

Theoretically, it is rather difficult to derive a closed-form solution to the optimal control problem, since: 1) the state transition is explicitly time-dependent and 2) the control variable  $k_t$  is implicitly embedded in  $x_{k_t}$  in the state transition equation, and is constrained to only discrete values. In the literature, a common way to obtain an approximately optimal solution for such tasks is to discretize the continuous state space and apply numerical approximation methods in the finite state dynamic programming research [28], [29]. Following this line of thought, we discretize the state space into a finite set of values, and use the cumulative revenue from stage 1 to  $T \geq 1$

TABLE I  
PARAMETER SETTING OF EXAMPLE 1

Auction Parameters	Values	Algorithm Parameters	Values
# of Slots	6	# of Stages	100
Discount factor	0.7677	# of States	100
Per-click value	0.3094	$\varepsilon$	0.0001
Initial QS	0.8837		
CTR	[0.6365, 0.3951, 0.3603, 0.2098, 0.1609, 0.0771]		
Ad-Rank	[0.5895, 0.2816, 0.1753, 0.1632, 0.1104, 0.0571]		

to approximate the infinite sum of intertemporal revenue in all stages. This way, we can numerically determine the optimal strategy based on policy iteration.<sup>15</sup>

The key idea of policy iteration is the use of value functions to organize the search for the optimal policies. The optimal value function, denoted by  $V^*(q, t)$ , represents the maximum possible revenue that an advertiser with an immediate QS of  $q$  can obtain after stage  $t$ . Since our model is explicitly time-dependent, the policies will be optimized through iterative search in the discrete state-action-stage space. The optimal value function satisfies the Bellman optimality equation, and can be defined as follows:

$$V^*(q, t) = \max_{\substack{k_t \in \mathcal{K} \\ q_t = q}} \{ u(q_t, k_t) + \delta V^*(q_{t+1}, t+1) \} \quad t = 1, \dots \quad (5)$$

In our model with finite stages, actions and discretized states, policy iteration can be guaranteed to converge to the optimal policy in a finite number of iterations, and the number of iterations is upper-bounded by the total number of possible iterations [28], [30]. Since the policy iteration algorithm can strictly improve the policy in every iteration [30], there is no risk that it oscillates among multiple optimal policies. The detailed optimization algorithm is given as follows.

Using Algorithm 1, we can investigate an advertiser’s optimal strategy in various SSA scenarios. Below we present our key conclusion with an illustrative example.

*Example 1:* Without loss of generality, we use the auction scenario listed in Table I, in which the auction parameters are generated following normal distributions in order to cover all possible SSA auction scenarios with equal possibility. The vicious circle due to the CTR-QS correlation and advertiser’s downward rank adjustments can be observed in this example.

*Conclusion:* The optimal positioning strategy of an advertiser is to increase or decrease the rank monotonically.<sup>16</sup> The direction of the rank adjustment depends on the advertiser’s per-click value and the initial QS of the advertisement. Therefore, with advertisers monotonically adjusting their targeting ranks, a polarization trend is expected to emerge in SSA markets.

<sup>15</sup>Another kind of reinforcement learning algorithm, Q-leaning, can yield exactly the same optimal strategy. However, policy iteration converges to the optimal policy using less iterations, and we thus report our research using policy iteration.

<sup>16</sup>The positioning dynamics of monotonic rank adjustments can be observed from our experiments reported in Section VI with more than 1000 randomly generated scenarios.

**Algorithm 1** Positioning Strategy Optimization Based on Policy Iteration

**Input:** An SSA scenario (including CTR, ad-rank, discount factor, initial QS, and the advertiser’s per-click value)

**Output:** The maximum cumulative revenue and the optimal positioning strategy

1. %% Step 1: Initialization
2.     For each  $q \in \mathcal{Q}$  and  $t \in \mathcal{T}$
3.         Arbitrarily set  $V(q, t) \in \mathcal{R}$  and  $\sigma(q, t) \in \mathcal{K}$
4. %% Step 2: Policy Evaluation
5.     Repeat
6.          $\Delta \leftarrow 0$ ;
7.         For each  $q \in \mathcal{Q}$  and  $t \in \mathcal{T}$
8.              $v = V(q, t)$ ;
9.              $q' = (q * t + x_{\sigma(q,t)}) / (t + 1)$
10.              $V(q, t) \leftarrow u(q, \sigma(q, t)) + \delta * V(q', t + 1)$ ;
11.              $\Delta \leftarrow \max(\Delta, |v - V(q, t)|)$ ;
12.     Until  $\Delta < \varepsilon$  (a small positive number)
13. %% Step 3: Policy Improvement
14.     *PolicyStable*  $\leftarrow$  *True*;
15.     For each  $q \in \mathcal{Q}$  and  $t \in \mathcal{T}$
16.          $b \leftarrow \sigma(q, t)$ ;
17.          $\sigma(q, t) \leftarrow \operatorname{argmax}_{k \in [0, K]} [u(q, k) + \delta * V(q'', t + 1)]$ ,  
            where  $q'' = (q * t + x_k) / (t + 1)$ ;
18.         If  $b \neq \sigma(q, t)$ , then *PolicyStable*  $\leftarrow$  *false*;
19.     If *PolicyStable* = *false*, then goto 5.
20. %% Step 4: Algorithm Output
21.     Set  $q_1 =$  initial QS, and cumulative revenue  $u = 0$ ;
22.     For  $t = 1$  to  $T$
23.          $k_t = \sigma(q_t, t)$ ;
24.          $u = u + u(q_t, k_t)$ ;
25.          $q_{t+1} = (q_t t + x_{k_t}) / (t + 1)$ ;
26.     Output the maximum cumulative revenue  $u$ , and the optimal positioning strategy  $(k_1, k_2, \dots, k_T)$ .

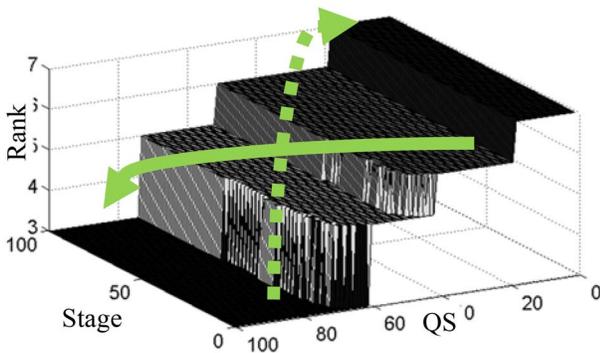


Fig. 4. Optimal strategy generated by Algorithm 1.

Fig. 4 presents the optimal action (i.e., targeting rank) at each state and stage in Example 1, derived from Algorithm 1. We can see an obvious characteristic of a “staircase-like” distribution in advertisers’ optimal actions. The width of each staircase along the QS dimension is determined by the advertiser’s per-click value. An advertiser with a higher per-click value will enjoy a wider staircase of better ranks, and thus are more likely to get a higher rank given a specific initial QS.

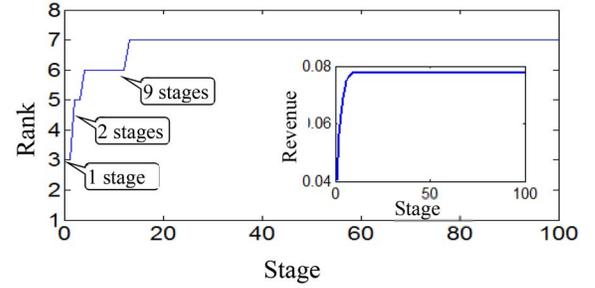


Fig. 5. Optimal sequence of slots and cumulative revenue.

Although the positioning dynamics and the optimal sequence of targeting slots are entirely determined by all variables listed as inputs in Algorithm 1, the comparison of the advertiser’s initial QS and the CTR of the optimal slot in stage 1 can be used as an indicator for predicting the direction of rank adjustments. For instance, the behavior dynamics of an advertiser in Fig. 4 can be depicted as follows: if the initial QS of the advertiser is lower than the CTR of the optimal slot in stage 1, the QS and rank will monotonically rise over stages, and the advertiser will “go downstairs” following the trajectory illustrated by the solid arrow, until stabilizing at the highest possible slot (e.g., the third highest slot in Fig. 4). Otherwise, the advertiser has to lower the rank due to a decreasing QS and “go upstairs” following the dotted arrow, and even quit the SSA auctions (i.e., the seventh dummy slot in Fig. 4). Fig. 5 shows the optimal sequence of the advertiser’s slot adjustment, and the curve of the cumulative revenue. As can be seen in Fig. 5, since the initial QS (e.g., 0.8837) is higher than the CTR of the optimal slot in stage 1 (e.g., 0.3603 of the third slot), the advertiser drops from the third slot directly to the fifth slot, then to the sixth slot, and finally quits the SSA auction.

It is worth noting that in SSA practice, many advertisers do not have sufficient information, expertise or even patience to identify the optimal positioning strategy as given by Algorithm 1. Alternatively, they typically use heuristic strategies such as the greedy and farsighted strategies to guide their position decisions. As such, one might ask whether these practically used positioning strategies will lead to monotonic rank adjustments and in turn the polarization trend in SSA markets. In the following sections, we will analyze the greedy and farsighted positioning strategies, aiming to help explain advertisers’ real-world behavior dynamics caused by the CTR-QS correlation in SSA auctions.

#### IV. ANALYSIS OF THE GREEDY STRATEGY

In practice, many skilled and highly price-sensitive advertisers prefer to keep monitoring their QSs, and make a decision about the optimal targeting slot in the present stage. This is especially prominent for myopic advertisers who value their immediate revenue the most. We call this kind of positioning strategy a greedy strategy. In this section, we show that, due to the CTR-QS correlation, advertisers using the greedy strategy will also monotonically adjust their targeting ranks, which may cause a polarization trend in the system-wide SSA markets.

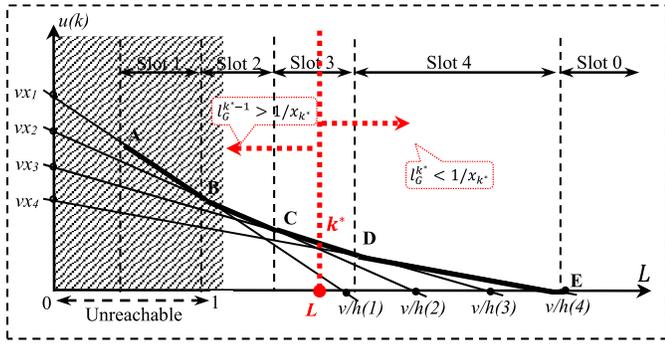


Fig. 6. Behavior dynamics of greedy advertisers. An illustrative example.

**Definition 1: Greedy Strategy.** A positioning strategy is called a greedy strategy if

$$\sigma_G^*(q_t, t) = \operatorname{argmax}_{k_t \in \mathcal{K}} u(q_t, k_t), \quad t = 1, 2, \dots \quad (6)$$

Obviously, a greedy advertiser always selects the optimal slot to maximize the immediate revenue in each stage, without considering the potential influence of current actions on future states.<sup>17</sup>

For notational simplicity, in subsequent analyses, we define the inverse QS (IQS for short) of an advertisement by  $L = 1/q$ , and so we have  $L > 1$ . Similarly,  $\sigma_G^*(L, t)$  denotes the optimal slot for an advertiser with a QS of  $q_t = 1/L$ . An advertiser's revenue in slot  $k$  can thus be transformed into a linear function as follows:

$$u(L, k) = -h(k) x_k L + v x_k. \quad (7)$$

The above equation represents a family of  $K$  revenue functions with  $k$  ranged from 1 to  $K$ , each specifying the advertiser's revenue in one slot. Clearly, these revenue functions have incremental slopes since  $-1 < -h(1)x_1 < \dots < -h(K)x_K$ , and have decreasing intercepts since  $v x_1 > \dots > v x_K$ . Fig. 6 plots these revenue functions with an illustrative example of  $K = 4$ . For each possible value of IQS, a greedy advertiser should choose the slot with the maximum revenue, which corresponds to the boldfaced segments A-B-C-D-E. These segments constitute the optimal path for a greedy advertiser's slot adjustments over time.

Below we will determine the optimal path of a greedy advertiser's slot adjustments. For each pair of slots  $(k, j)$ , we can compute a matrix defined as follows:

$$LM(k, j) = \frac{v(x_k - x_j)}{h(k)x_k - h(j)x_j}, \quad k, j \in [1, K], \quad k \neq j. \quad (8)$$

Geometrically,  $LM(k, j)$  is the value of the IQS corresponding to the intersection of two revenue functions, namely  $u(L, k)$  and  $u(L, j)$ . Using this  $LM$  matrix, below we present the optimal path, the behavioral dynamics, and the stable state of a greedy advertiser's slot adjustments, with three theorems.

<sup>17</sup>Note that our greedy positioning strategy differs significantly with the greedy bidding strategy defined by Cary *et al.* [16], in which the CTR-QS correlation is not considered and advertisers always choose the optimal bid (instead of targeting rank) to maximizing their revenue in each stage, assuming other advertisers' bids remain fixed to the values in the previous stage.

We begin our analyses with a lemma concerning the monotonic convergence of an advertiser's QS.

**Lemma 1:** The QS of an advertiser staying in slot  $k$  converges monotonically to  $x_k$ , or  $\lim_{t \rightarrow \infty} q_t = x_k$ .

*Proof:* We can easily derive the following equations from the state transition equation (4):

$$q_t t = q_{t-1} (t-1) + x_k = \dots = q_1 + (t-1)x_k. \quad (9)$$

Therefore, we have  $\lim_{t \rightarrow \infty} q_t = \lim_{t \rightarrow \infty} q_1 + (t-1)x_k/t = x_k$ . ■

Lemma 1 indicates that due to the CTR-cumulative effect on the QS, if advertisers keep staying in one specific slot, their QS will monotonically converge to the CTR of that slot.

**Theorem 1:** For any  $L > 1$  and  $\sigma_G^*(L, t) = k^*$ , a greedy advertiser's next targeting slot in the upward rank adjustments is  $j^* = \operatorname{argmax}_{j < k^*} LM(k^*, j)$ , while the next targeting slot in the downward rank adjustments is  $j^* = \operatorname{argmin}_{j > k^*} LM(k^*, j)$ .

*Proof:* For any arbitrary  $L > 1$  and  $\sigma_G^*(L, t) = k^*$ , let  $j_1^* = \max\{j | LM(k^*, j) < L\}$ , and  $j_2^* = \min\{j | LM(k^*, j) > L\}$ . Geometrically, the intersections of utility functions  $u(L, j_1^*)$  and  $u(L, j_2^*)$  with  $u(L, k^*)$  are the nearest ones to the IQS  $L$  on both sides, respectively. Because of the monotonic increasing slopes and decreasing intercepts of advertisers' utility functions, there must be  $j_1^* < k^*$  and  $j_2^* > k^*$ , and thus  $j_1^* = \operatorname{argmax}_{j < k^*} LM(k^*, j)$ ,  $j_2^* = \operatorname{argmin}_{j > k^*} LM(k^*, j)$ . The intersections are  $LM(k^*, j_1^*)$  and  $LM(k^*, j_2^*)$ , respectively.

We first prove the next targeting slot in advertisers' upward rank adjustments. For any arbitrarily small  $\epsilon > 0$ , since  $u(L, k^*) = u(L, j_1^*)$  when  $L = LM(k^*, j_1^*)$ , and  $h(j_1^*)x_{j_1^*} > h(k^*)x_{k^*}$ , we have

$$\begin{aligned} u(LM(k^*, j_1^*) - \epsilon, j_1^*) &= u(LM(k^*, j_1^*), j_1^*) + \epsilon h(j_1^*)x_{j_1^*} \\ &= u(LM(k^*, j_1^*), k^*) + \epsilon h(j_1^*)x_{j_1^*} \\ &> u(LM(k^*, j_1^*), k^*) + \epsilon h(k^*)x_{k^*} \\ &= u(LM(k^*, j_1^*) - \epsilon, k^*). \end{aligned} \quad (10)$$

Because  $\sigma_G^*(L, t) = k^*$  when  $LM(k^*, j_1^*) < L < LM(k^*, j_2^*)$ , we can conclude from (10) that when  $L = LM(k^*, j_1^*) - \epsilon$ , we have  $\sigma_G^*(L, t) = j_1^*$ . Therefore, a greedy advertiser's optimal action is targeting the higher slot  $j_1^* < k^*$  whenever the IQS drops below  $LM(k^*, j_1^*)$ .

Analogically, we can prove that  $u(LM(k^*, j_2^*) + \epsilon, j_2^*) > u(LM(k^*, j_2^*) + \epsilon, k^*)$ , so that advertisers should target the lower slot  $j_2^* > k^*$  when  $L > LM(k^*, j_2^*)$ . ■

Using Theorem 1, we can recursively determine the optimal path for a greedy advertiser's slot adjustments, starting from the top slot. The optimal path can be defined as follows.

**Definition 2: The Greedily Optimal Path.** The greedily optimal path  $P = (k_G^1, k_G^2, \dots, k_G^{|P|})$  is an increasing sequence of slots, where

$$k_G^i = \operatorname{argmin}_{j > k_G^{i-1}} LM(k_G^{i-1}, j), \quad k_G^1 = 1, \quad i, j \in \mathcal{K}. \quad (11)$$

We also compute a vector  $l = (l_G^1, l_G^2, \dots, l_G^{|P|})$ , where

$$l_G^i = \min_{j > k_G^{i-1}} LM(k_G^{i-1}, j), \quad \forall i, j \in \mathcal{K}. \quad (12)$$

Each element  $l_G^i$  denotes the corresponding value of the advertiser's IQS for a slot switching from  $k_G^i$  to  $k_G^{i+1}$ , and  $l_G^{PI} = v/h(K)$ . Let  $l_G^0 = 0$ .

It is worth noting that greedy advertisers might not traverse all slots along their optimal path, so that  $|P| = K + 1$ . The reason is twofold. First, a slot, say  $k$ , might not be an optimal action for any possible IQS, or formally,  $\forall L, \sigma_G^*(L, t) = k$  does not hold (e.g., the sixth slot in Example 2). Second, some slots cannot be reachable since advertisers might not be able to reach the corresponding IQSs (e.g., the 1st slot illustrated in Fig. 6). Thus, the realized optimal path, denoted by  $P^*$ , is only a subset of the optimal path  $P$ . The maximum length of the optimal path, i.e.,  $|P| = K + 1$ , only corresponding to an extreme scenario in which, due to the continuously deteriorating QS, an advertiser has to drop from the top slot, one by one, to the bottom slot, and finally quit the auctions.

*Theorem 2:* For any arbitrary  $L > 1$  and  $\sigma_G^*(L, t) = k^*$ , a greedy advertiser will raise the rank if  $l_G^{k^*-1} > 1/x_{k^*}$  and lower the rank if  $l_G^{k^*} < 1/x_{k^*}$ .

*Proof:* We first prove the case when an advertiser raises the slot. For any arbitrary  $L > 1$  and  $\sigma_G^*(L, t) = k^*$ , we have  $L \geq l_G^{k^*-1}$ . So if  $l_G^{k^*-1} > 1/x_{k^*}$ , then  $L > 1/x_{k^*}$ . As such, an advertiser staying on slot  $k^*$  will keep improving the QS and we have  $\lim_{t \rightarrow \infty} q_t = x_{k^*}$  according to Lemma 1. Since  $1/x_{k^*} < l_G^{k^*-1}$ , there must exist one stage  $t' > 0$  so that  $1/q_t < l_G^{k^*-1}$  holds for all  $t > t'$ . According to Theorem 1, we have  $\sigma_G^*(1/q_t, t) = k_G^{k^*-1}$ , where  $k_G^{k^*-1} < k^*$ . As such, a greedy advertiser can raise the slot to at least  $k_G^{k^*-1}$  when  $l_G^{k^*-1} > 1/x_{k^*}$ .

Analogously, when an advertiser lowers the slot, we have  $L \leq l_G^{k^*} < 1/x_{k^*}$ . The QS of an advertiser staying on slot  $k^*$  keeps decreasing and we have  $\lim_{t \rightarrow \infty} q_t = x_{k^*}$ . Since  $1/x_{k^*} > l_G^{k^*}$ , there must exist  $t' > 0$  so that  $1/q_t > l_G^{k^*}$  for all  $t > t'$ . Thus, we have  $\sigma_G^*(1/q_t, t) = k_G^{k^*+1}$ , where  $k_G^{k^*+1} > k^*$ , according to Theorem 1. Therefore, a greedy advertiser has to lower the slot to at most  $k_G^{k^*+1}$  when  $l_G^{k^*} < 1/x_{k^*}$ . ■

As illustrated in Fig. 6, Theorem 2 shows that if  $l_G^{k^*-1} > 1/x_{k^*}$ , a greedy advertiser can keep improving the QS and target a higher slot with more revenue realized. Otherwise, if  $l_G^{k^*} < 1/x_{k^*}$ , a greedy advertiser suffers from a decreasing QS and thus has to lower the slot with less revenue realized, and even finally quit the SSA auctions (when  $L > v/x_K$ ).

We can draw the following important conclusion from Lemma 1 and Theorems 1 and 2: the optimal strategy for greedy advertisers is to increase or decrease their ranks monotonically along the optimal path. The direction of a greedy advertisers' rank adjustment depends on their per-click value and the initial QS of the advertisement. Note from Fig. 5 that the intersections of revenue functions with both axes are linear functions of advertisers' per-click values. As such, advertisers' values actually serve as a scale controller that can proportionately zoom in or out the landscape of all revenue functions. That is, *ceteris paribus*, advertisers with higher per-click values will enjoy a proportionately higher  $LM$  matrix, and hence a higher opportunity of raising their ranks. On the contrary, advertisers with lower per-click

values are more likely to be forced to target lower slots with their QSs further decreasing. Therefore, due to the cumulative effect of the CTR on the QS, such divergent behavior dynamics is expected to cause a polarization trend in the SSA markets.

*Theorem 3:* A greedy advertiser will stabilize in a slot  $k^*$ , which is the first slot along the optimal path satisfying

$$l_G^{k^*-1} < \frac{1}{x_{k^*}} < l_G^{k^*}. \quad (13)$$

We define the slot  $k^*$  as an absorbing slot, and the corresponding IQS value  $L = 1/x_{k^*}$  as a stable state.

*Proof:* We consider an advertiser in slot  $k$ . According to Theorem 2, that advertiser will jump to a higher slot if  $l_G^{k-1} > 1/x_k$ , and drop to a lower slot if  $l_G^k > 1/x_k$ . In both cases, the advertiser will not stabilize in the slot  $k$ . We denote the first slot that satisfies (13) during the advertiser's slot adjustments along the optimal path as  $k^*$ . Thus, once the IQS falls into the interval of  $(l_G^{k^*-1}, l_G^{k^*})$  in a stage  $t'$ , the advertiser will find it optimal to target slot  $k^*$ , and the IQS converges to  $1/x_{k^*}$  according to Lemma 1. Since  $1/x_{k^*}$  also falls into the interval of  $(l_G^{k^*-1}, l_G^{k^*})$ , we can easily conclude that  $\sigma_G^*(L, t) = k^*$  holds for any subsequent stages  $t > t'$ , which indicates that the advertiser stabilizes in and will never leave the slot  $k^*$ . ■

Theorem 3 determines the long-run stable state of a greedy advertiser's slot adjustment. It can be easily proven that at least one absorbing slot exists if advertisers monotonically increase their ranks (e.g., the third slot in Example 2).<sup>18</sup> However, absorbing slots might not exist when advertisers decrease their ranks. In this case, advertisers will finally find that the optimal action is to quit the SSA auctions.

We illustrate the above findings of greedy slot adjustments with the following example.

*Example 2:* We use the following auction scenario. Virtuous circle due to the CTR-QS correlation and the advertiser's upward rank adjustments can be observed in this example.

In this auction scenario, the  $LM$  matrix can be computed using (8), and is shown below in (14). Using (11) and (12), we can recursively compute the optimal path for greedy advertisers' bid adjustments, i.e.,  $P = (1, 2, 3, 4, 5, 7, 8)$ , and the corresponding values of IQS for the slot switching, i.e.,  $l = (0.20, 0.67, 6.46, 16.23, 36.31, 75.12, 469.82)$ . Note that  $P$  is the set of the optimal slots for all possible QSs, and thus the fact that the sixth highest slot is not included in  $P$  indicates that the advertiser will never choose this slot as the optimal response to the current QS. Meanwhile, since we assume  $q \in (0, 1)$  and the IQS  $L = 1/q$ , we have  $L > 1$ . Therefore, the top two slots will be unreachable because

<sup>18</sup>Note that in advertisers' upward rank adjustments,  $1/x_{k^*}$  decreases monotonically to 1 and  $l_G^{k^*}$  decreases to 0. Since  $1/x_K < l_G^K$  (otherwise advertisers will quit the auctions), if all  $k^* > 1$  do not hold for (13), then  $k^* = 1$  must be a solution to it, so that there exists at least one solution.

$$l_G^1 = 0.20 < 1 \text{ and } l_G^2 = 0.67 < 1$$

$$LM = \begin{pmatrix} 0 & \mathbf{0.20} & 0.35 & 0.49 & 0.69 & 0.80 & 0.89 & 1.00 \\ 0 & 0 & \mathbf{0.67} & 1.08 & 1.65 & 1.97 & 2.24 & 2.54 \\ 0 & 0 & 0 & \mathbf{6.46} & 9.98 & 12.18 & 13.68 & 15.72 \\ 0 & 0 & 0 & 0 & \mathbf{16.23} & 20.51 & 22.49 & 26.23 \\ 0 & 0 & 0 & 0 & 0 & 40.15 & \mathbf{36.31} & 43.94 \\ 0 & 0 & 0 & 0 & 0 & 0 & 32.76 & 46.32 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & \mathbf{75.12} \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}. \quad (14)$$

We can further verify that there exists a unique absorbing slot, i.e., the third slot, and the stable state will be  $L = 1/x_3 = 1.61$ . Since the initial IQS is  $L = 200$ , the realized optimal path will be  $P^* = \{3, 4, 8\}$ , which indicates that the advertiser will start from the bottom 8th slot, increase the rank to the fourth slot, and finally stabilize in the third slot.

## V. ANALYSIS OF THE FARSIGHTED STRATEGY

In SSA practice, big-brand advertisers might want to show their advertisements in more prominent slots without considering whether the payments in those slots are economically justifiable or not. The main objective of this strategy is to create, maintain or enhance their brand awareness, and also effectively suppress their competitors [31]. Currently, various kinds of services or tools in major SSA platforms have been developed to help advertisers manage their bids automatically and maintain some specific positions (e.g., the top slot or top three slots) for their advertisements. At first glance, such positioning strategy will possibly lead to unaffordable high CPCs and payments. Due to the CTR-cumulative effect on the QS, however, targeting prominent slots with high CTRs might be quite effective in SSA practice. The secret behind this seemingly irrational strategy lies in that, although advertisers targeting higher slots might possibly suffer from revenue loss in the initial stages, the increasing QSs due to higher CTRs will compensate their loss with decreasing CPCs and possibly satisfying revenue in the long run.

In this paper, we investigate a rational implementation for this strategy: an advertiser always makes an optimal decision to find and stay in one slot, which can maximize the discounted intertemporal sum of revenue in all stages. We call it a farsighted strategy. From the modeling perspective, a farsighted positioning strategy can be viewed as a stationary policy for the optimal control model in Section II with the same action in all states and stages, and can be formally defined as follows.

*Definition 3: Farsighted Strategy.* A positioning strategy is called a farsighted strategy if it satisfies

$$\sigma_F^*(q_t, t) = \operatorname{argmax}_{k \in \mathcal{K}} \sum_{t=1}^{\infty} \delta^{t-1} u(q_t, k). \quad (15)$$

Let  $\Sigma = \{\mu_0, \mu_1, \dots, \mu_K\}$  be the set of all stationary strategies, where  $\mu_k = \{k, k, \dots\}$  and  $|\Sigma| = K + 1$ . The optimal slot for a farsighted strategy is denoted by  $k_F^*$ . Below

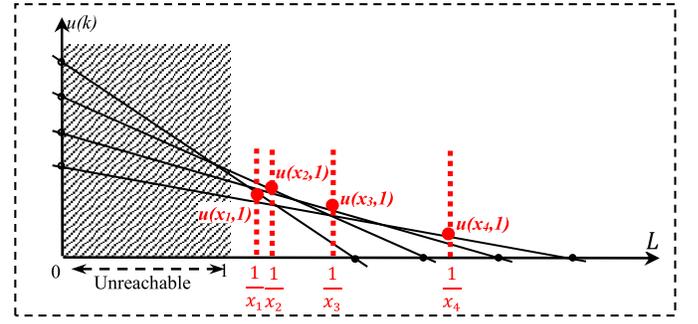


Fig. 7. Behavior dynamics of farsighted advertisers. An illustrative example.

we will investigate the behavior dynamics of advertisers using farsighted strategies.

*Theorem 4:* The optimal slot for an advertiser using the farsighted strategy is

$$k_F^* = \operatorname{argmax}_{k \in \mathcal{K}} [vx_k - h(k)]. \quad (16)$$

*Proof:* For any arbitrary state  $q_t \in \mathcal{Q}$  and a stationary policy  $\mu_k \in \Sigma$ , the value function will be

$$V(q_t, t) = u(q_t, k) + \delta V(q_{t+1}, t+1) \quad \text{where } t = 1, 2, \dots \text{ and } \mu_k = \{k, k, \dots\}. \quad (17)$$

According to Lemma 1, if advertisers use the stationary strategy  $\mu_k$ , then we have  $\lim_{t \rightarrow \infty} q_t = x_k$ , and thus the future revenue on slot  $k$  will be

$$\lim_{t \rightarrow \infty} u(q_t, k) = u(x_k, k) = vx_k - h(k). \quad (18)$$

Since the sequences of states  $q_t$  and revenue  $u(q_t, k)$  are monotonic, the discounted sum of revenue and value function will thus converge steadily. Further, due to the continuity of  $u(q_t, k)$ ,  $V(q_t, t)$ , and  $q_{t+1}$  on  $q_t$ , we have

$$\begin{aligned} \lim_{t \rightarrow \infty} V(q_t, t) &= \lim_{t \rightarrow \infty} [u(q_t, k) + \delta V(q_{t+1}, t+1)] \implies V(x_k) \\ &= u(x_k, k) + \delta V(x_k) \implies V(x_k) = \frac{u(x_k, k)}{1-\delta}. \end{aligned} \quad (19)$$

The optimal slot for an advertiser using a farsighted strategy can thus be computed as follows:

$$\sigma_F^*(q_t, t) = k_F^* = \operatorname{argmax}_{k \in \mathcal{K}} V(x_k) = \operatorname{argmax}_{k \in \mathcal{K}} [vx_k - h(k)]. \quad (20)$$

So we have proved Theorem 4.  $\blacksquare$

Note that  $u(x_k, k)$  is the future revenue of advertisers in a slot  $k$ . Therefore, Theorem 4 indicates that farsighted advertisers will always target the slot that maximizes their future revenue. Fig. 7 illustrates the slot adjustment dynamics with an example, in which we have  $k_F^* = 2$ . The optimal farsighted strategy can be easily computed by traversing the strategy space  $\Sigma$ , since there are only finite  $K + 1$  stationary strategies.

*Theorem 5:* The slot  $k$  is profitable to an advertiser using the farsighted strategy if

$$v > \frac{h(k)}{x_k}. \quad (21)$$

TABLE II  
PARAMETER SETTING OF EXAMPLE 2

Auction Parameters	Values	Auction Parameters	Values
# of Slots	8	Per-click value	0.5168
Discount factor	0.98	QS	0.005
CTR	[0.8531, 0.7622, 0.6228, 0.5217, 0.3787, 0.3015, 0.2335, 0.1600]		
Ad-Rank	[0.4215, 0.1611, 0.0247, 0.0140, 0.0072, 0.0058, 0.0029, 0.0011]		

*Proof:* For any arbitrary  $k \in [1, K]$ , the slot  $k$  is profitable for an advertiser if cumulative revenue is positive, or formally

$$\sum_{t=1}^{\infty} \delta^{t-1} u(q_t, k) > 0 \iff V(x_k) = \frac{u(x_k, k)}{1 - \delta} > 0$$

$$\iff vx_k - h(k) > 0 \iff v > \frac{h(k)}{x_k}. \quad (22)$$

So we have proved Theorem 5.  $\blacksquare$

Theorem 5 indicates that the profitability of a slot to a farsighted advertiser depends only on the advertiser's per-click value, provided the ad-ranks and CTRs of all slots are given as constants. Theorem 5 also presents the minimum per-click values of advertisers competing for each advertisement slot. We can draw a conclusion from Theorem 5 that since  $h(k)/x_k$  differs from each slot, maintaining the advertisements in the top slot or top three slots might not always be an optimal choice for farsighted advertisers, especially for small-to-medium advertisers with relatively lower per-click values. The reason lies in that due to the discounting effect, the increasing QS and revenue in the later stages cannot compensate the high CPCs in the initial stages.

It is worth noting that the farsighted strategy is particularly useful for patient advertisers with large discount factors, who care less on immediate gains but pay more attention to the increasing future revenue due to the CTR-cumulative effect on the QS. On the contrary, short-sighted advertisers with low discount factors will be more inclined to attach more importance on immediate revenue and thus use the greedy positioning strategy.

*Example 3:* We illustrate the dynamics using the SSA scenario in Table II, except that the initial QS is randomly set to be 0.3679 here. Obviously, the optimal slot for a farsighted advertiser is

$$k_F^* = \operatorname{argmax}_{k \in \mathcal{K}} [vx_k - h(k)] = 3. \quad (23)$$

The cumulative revenue of the advertiser on the 3rd slot and the 1st slot is shown in Fig. 8.

We can see from Fig. 8(a) that the cumulative revenue of a farsighted advertiser on the optimal 3rd slot will increase monotonically to the maximum revenue that can be obtained among all stationary policies.<sup>19</sup> On the other hand, if targeting the 1st slot, we can see from Fig. 8(b) that the advertiser does not make profits in the initial stages. Although positive revenue can be obtained since the 12th stage, the discounted sum of

<sup>19</sup>The maximum revenue can be verified by comparing the revenue obtained in all eight slots and zero (i.e., not entering the auction).

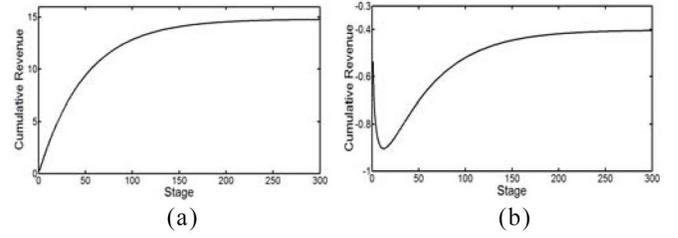


Fig. 8. Advertisers' cumulative revenue on the 1st and 3rd slot. (a) Cumulative revenue on the 3rd slot. (b) Cumulative revenue on the 1st slot.

revenue in the later stages cannot compensate the loss in the initial stages, and the cumulative revenue in the long-term will be negative. As such, the top slot is not an optimal slot for the farsighted advertiser in this SSA scenario.

The farsighted strategy is particularly effective for big-brand advertisers that target prominent positions and those "lazy" advertisers that seldom change their bids.<sup>20</sup> With the help of bidding software, maintaining a specific slot determined by the farsighted strategy will be a good alternative for these advertisers. As long as the position is maintained for a sufficiently long period of time (e.g., after the revenue can compensate the loss in initial stages), the farsighted strategy will be profitable, although might not be optimal in dynamic environments.

Unlike the optimal and greedy strategies, the farsighted strategy does not lead to divergent dynamics in advertisers' slot adjustments, and thus does not directly contribute to the polarization trend in SSA markets. However, the farsighted strategy can help accelerate the polarization trend in an indirect fashion. Due to the CTR-cumulative effect on the QS, big-brand advertisers that can afford the initial revenue loss will now be more inclined to aggressively bid for the top slots, so as to maximize their brand awareness while enjoying improving QSs and revenue. On the contrary, small advertisers can only compete for lower slots with their QSs being unchanged or even worsened. Thus, with the gap of QSs between big-brand and small advertisers widening, small advertisers have to afford much higher CPCs to defeat the big-brand advertisers. As a direct consequence, the top slots on search results pages will be always occupied by big-brand advertisers, and this helps accelerate the polarization trend in SSA markets.

## VI. DISCUSSION

In this section, we present some related discussions concerning the revenue performance of the above investigated positioning strategies, their practical feasibility in SSA auctions with incomplete information settings, and potential solutions and policy suggestions to deal with the CTR-QS correlation and the resulting polarization trend in SSA markets.

### A. Revenue Performance of Positioning Strategies

As aforementioned, advertisers in practice typically cannot identify the optimal positioning strategy in their slot

<sup>20</sup>An empirical research indicates that about 70 percent of advertisers change their bids on weekly, monthly or even quarterly bases. Please see <http://www.seroundtable.com/archives/022226.html>

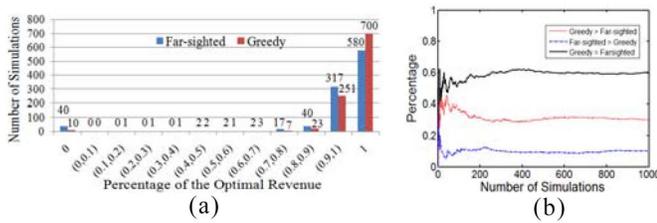


Fig. 9. Experimental results of the strategy comparison. (a) Greedy and farsighted strategies versus the optimal strategy. (b) Greedy strategy versus farsighted strategy.

adjustments, and instead are more likely to use the greedy or farsighted strategies. As such, two natural questions to ask are which one of these two alternative strategies is better, and how good is their revenue performance compared with the optimal strategy. In this subsection, we want to answer these questions by quantitatively investigating and comparing the revenue performance of these strategies using simulation experiments.

In our experiments, we generate 1000 SSA auction scenarios with their parameters stochastically drawn from specific distributions, and numerically determine the advertiser's cumulative revenue derived from the optimal, greedy and farsighted strategies, respectively. This way we can quantitatively measure the revenue performance of the greedy and farsighted strategies, using the optimal strategy as a baseline.

The experimental parameters of the SSA auction scenarios are configured as follows. Similarly to the existing SSA simulations, the advertiser's per-click value and initial QS follow normal distributions  $v \sim N(0.5, 0.2)$  [17] and  $q_1 \sim N(0.7, 0.3)$ ,<sup>21</sup> respectively. The ad-rank vector is drawn from a uniform distribution on the interval of  $[0, 1]$ , and sorted in decreasing order. Following [32], we set the CTR of the  $k$ th highest slot to be  $x_k = x_1/\gamma^{k-1}$  to capture the feature of exponential decay of user attention, where  $\gamma = 1.428$  and  $x_1$  is uniformly distributed in the interval of  $[0, 1]$ . Moreover, we use the cumulative revenue in the first 100 stages to approximate the total revenue in the infinitely repeated SSA auctions, and set the discount factor to be  $\delta = 0.95$ . With this discount factor, the total contribution from stage 101 to infinity is 0.006, and thus the history thereafter can be assumed negligible [17]. Finally, the number of advertisement slots is randomly set from 1 to 10 in each scenario.

Fig. 9 presents the experimental results concerning the revenue performance of three strategies. We can see from Fig. 9(a) that both the greedy and farsighted strategies perform quite well compared with the optimal strategy. More specifically, in 700 and 580 out of the 1000 auction scenarios, the greedy and farsighted strategies perform equally well to the optimal strategy, and both strategies lead to more than 90% of the maximum possible revenue in at least 951 and 897 scenarios, respectively. We also observe in 10 and 40 scenarios that the greedy and farsighted strategies lead to zero revenue, respectively (i.e., the advertiser does not

enter the auctions), whereas the optimal strategy results in a positive revenue. These results indicate that both the greedy and farsighted strategies can be used as good alternatives of the optimal strategy in online SSA markets. Fig. 9(b) shows the percentage of scenarios in which each strategy outperforms the other. We can see that two strategies perform equally well in about 60% of scenarios, while the greedy strategy performs better in about 30% of scenarios. These experimental results show that the greedy strategy is slightly better than the farsighted strategy. However, it is obvious in SSA practice that each kind of strategy has its own merits and demerits. For instance, the greedy strategy typically needs advertisers to spend more time and expertise in monitoring the QSs of their advertisements, while the farsighted strategy has the advantage of being easy to be implemented but is less suitable to highly dynamic SSA markets. As such, advertisers should leverage their marketing objectives and constraints in choosing a better positioning strategy.

### B. Positioning Strategies in Incomplete Information Settings

One assumption of our research is that advertisers need complete information about their own QS, as well as the ad-rank and CTR of each slot. This complete information assumption is reasonable in practice. In online SSA platforms, it is very possible for skilled and highly price-sensitive advertisers to come up with estimations for these variables [4], [11], [14].

Even if the variables cannot be accurately computed, we argue that advertisers can also derive an approximately optimal positioning strategy, and monotonically adjust their targeting ranks. First, advertisers typically do not know the precise values of their own QS that are used in SSA platforms. Instead, they are released some designations representing the quality levels of their advertisements. For example, Google AdWords reports advertisers' QSs using integers ranged from 1 to 10, with 1 being the poorest and 10 being the best. These designations can serve as rough estimates of advertisers' precise QS values, and a coarse-grained discretization of the state space (e.g., with ten states) in our algorithm to identify an approximately optimal strategy. Second, using the estimated QS, advertisers can come up with an estimation of the ad-rank in each slot by experimenting on the SSA markets through shifting their own bids and submitting dummy search requests to test the rank changes. Besides, the CTR of each slot can be easily estimated using online services and tools, or advertisers' own historical click-through data. As such, using our algorithm and analyses, advertisers can thus obtain an approximate implementation of the optimal, greedy, and farsighted strategies, and monotonically adjust their ranks for maximum revenue.

When all the above variables cannot be estimated due to constraints of advertisers' time, cost, expertise, or even patience, we argue that advertisers will also adjust their ranks monotonically, although they might not be able to identify the optimal strategy. Note that the direction of advertisers' rank adjustments depends only on their own per-click values and QSs. Thus, whether an advertiser should increase or decrease the rank in the next stage is determined only by his

<sup>21</sup>In major SSA platforms such as Google Adwords, the default QS of a new advertisement will be 7, which can be mapped to 0.7 in the interval of  $[0, 1]$  in our simulation.

or her QS, which might be unobservable by advertisers but can be directly inferred from the observable CPC in incomplete information settings. When QS of the advertisers is lower than the CTR of their slot, the QS will increase according to Lemma 1, and *ceteris paribus*, the CPC will be observed to keep decreasing. As a result, the advertiser's optimal action in the following stages is to target a higher slot, or at least keep the current position. The reverse can be deduced analogously. Therefore, although it is hard for advertisers to identify the optimal sequence of ranks in all stages, their rank adjustments are also monotonic, using the observable CPC to infer the latent variables such as QS, ad-rank, and CTR.

### C. Potential Solutions and Policy Suggestions

From a business perspective, the CTR-QS correlation and the resulting polarization may pose a potential threat to the SSA ecosystem. On one hand, as a direct consequence of the correlation, QS will be a biased measurement for Web search engines to evaluate the quality of sponsored advertisements, since big-brand advertisers can “buy” better QSs by aggressively bidding for prominent positions, especially the top three positions. Moreover, since inferior advertisements can easily get high QSs and appear in top positions with aggressive bids, the experience of search users might be reduced in the long run. On the other hand, due to the divergent dynamics illustrated in Fig. 1, big-brand advertisers may occupy prominent positions with high bids but lowered CPCs, which may reduce the profitability and effectiveness of SSA platforms in the long run.

Therefore, in order to help maintain the overall stability, profitability, and effectiveness of the SSA ecosystem, Web search engine companies must structure their policy to suppress the polarization trend caused by the CTR-QS correlation. Clearly, the most effective way toward this end is to break the virtuous and vicious circles depicted in Fig. 1 by eliminating or at least downplaying the causal relationship between the adjacent variables. We here discuss two potential solutions for this purpose.

First, Web search engine companies are advised to set a lowered weight for the historical CTR in the QS measurements. This way, we can directly reduce the CTR-QS correlation and alleviate the resulting polarization trend. The reason lies in that although advertisers are still willing to bid high for prominent positions and more clicks, the increased CTR can no longer help improve their QSs and CPCs. As such, advertisers will no more be motivated to further increase their bids for higher positions, and the divergent positioning dynamics can thus be avoided.<sup>22</sup> However, it is worth noting that CTR serves as an important objective evaluation for an advertisement's QS from search users. Setting a zero weight for historical CTR will increase the subjectivity of the QS since the other factors used in calculating QSs, such as landing page experience or keyword-query relevance, are determined largely by search engines and are less transparent to advertisers. Therefore, from

the perspective of the SSA ecosystem, Web search engine companies should maintain a lowered but nonzero weight for the historical CTR in their QS measurements.

Second, Web search engine companies can also normalize their CTR measurements and remove the positional bias from CTR. In SSA auctions, empirical CTR can be factorized into an advertiser-specific effect and a positional effect [33]. Removing the positional bias from CTR will break the causal effect between rank and CTR in Fig. 1, so that advertisers bidding for high positions will not get an increased CTR and in turn QS. Thus, the divergent positioning dynamics and polarization trend can be effectively avoided. In the literature, click-through behavior in Web search engines has been intensively studied [34], [35], and various approaches and algorithms have been proposed to help estimate the position-normalized CTR [36]–[39]. In SSA practice, however, it is still controversial whether or not most Web search engines have been aware of this positional bias on CTR and have taken actions to remove this bias (e.g., via normalization). For instance, Google AdWords still calculates the QS using the historical CTR defined in a traditional way as an empirical impression-to-click ratio.<sup>23</sup> It has been empirically observed that higher positions will lead to higher CTRs [5], and in turn higher QSs.<sup>24</sup> This positional bias on CTR has also been empirically witnessed in SSA platforms such as Microsoft adCenter [39].

## VII. CONCLUSION

In this paper, we studied advertisers' QS-based positioning strategy in online SSA auctions, with a focus on the correlation between the QSs and historical CTRs of their advertisements, which has been overlooked in the existing research. Using a policy iteration-based algorithm, we numerically determined the optimal positioning strategy for advertisers' rank adjustments, and also theoretically analyzed the behavioral dynamics of advertisers using two alternative strategies, i.e., the greedy strategy and the farsighted strategy. We proved that both the optimal and greedy strategies will lead advertisers to monotonically adjust their ranks and thus may cause a polarization trend in SSA markets. Moreover, the farsighted strategy can help accelerate this polarization trend. We further numerically compared the revenue performance of these strategies, and proved that both the greedy and farsighted strategies can serve as good alternatives to the optimal strategy.

There are several limitations in this research that should be addressed in future work. First, we focus on the decision-making of a single exogenous advertiser, and do not take into consideration the game-theoretic interactions among competing advertisers. As such, the ad-rank in each slot is assumed static in our model. However, we argue that this lack of game-theoretic analysis will not undermine our technical contributions. In SSA practice, many advertisers choose the optimal positions simply based on their own revenue, and do not take other advertisers' strategies into consideration due to the lack

<sup>22</sup>Advertisers' bidding and positioning dynamics in SSA auctions without the CTR-QS correlation have been thoroughly characterized by the existing research [3], [4], [14], [15].

<sup>23</sup><http://support.google.com/adwords/bin/answer.py?hl=en&answer=107955>

<sup>24</sup><http://www.clicks2customers.com/c2cblog/the-relationship-between-quality-score-and-click-through-rate-by-position.html>

of information, expertise or even patience. Our model and proposed strategies can offer meaningful insights for these advertisers. In future work, we plan to extend this paper by studying the equilibrium bidding and positioning strategies in repeated SSA auction games with CTR-QS correlation. Second, as advertisers can use the observable ranks, bids and CPCs to infer such latent variables as QSs and ad-ranks, we plan to use partially observable Markov decision process to model advertisers' rank adjustment process, and investigate the optimal positioning strategies in incomplete information settings.

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