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Adaptive neuro-fuzzy sliding mode control guidance law with impact angle constraint

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Abstract: This study presents a guidance law to intercept non-manoeuvring targets at a desired impact angle. The desired impact angle, defined in terms of a desired line-of-sight angle, is achieved by selecting the missile's lateral acceleration to enforce the sliding mode on a sliding surface. Then, the authors use the Lyapunov stability theory to prove the stability of the proposed non-linear sliding surface. Furthermore, they introduce the adaptive neuro-fuzzy inference system (ANFIS) to adaptively update the additional control command and reduce the high-frequency chattering of sliding mode control (SMC). The proposed guidance law, denoted ANFSMC guidance law with impact angle constraint, combines the SMC methodology with ANFIS to enhance the robustness and reduce the chattering of the system. The effectiveness of the ANFSMC guidance law is also verified by the numerical simulations.

1 Introduction

As optical interference technology and stealth technology have been greatly developed and progressed, the modern war becomes more harshly, intensely and complexly. Precision guidance of weapon systems is a computationally and conceptually demanding problem. In general, guidance laws allow the missiles to intercept the targets, such as ships or ballistic missiles at a zero terminal miss distance [1]. However, zero miss distance does not always guarantee the fulfillment of guidance laws. Impact angle control guidance laws aim to intercept the targets at a designed impact angle [2, 3]. It is well known that the main objective of the proportional navigation guidance (PNG) law is to produce zero terminal miss distance and cannot expect an acceptable performance in terms of impact angle [4].

Despite its necessity and importance in actual engagement, the guidance law with zero miss distance and impact angle constraints was not actively reported by Kim and Grider [5]. They put forward optimal and suboptimal guidance laws with impact angle constraint for reentry vehicles in the vertical plane. Since the 1970s, the guidance laws with impact angle have been made a great progress. The main outcomes of guidance laws with impact angle constraint were focused on the optimal (OP) control, biased PNG and sliding mode control (SMC).

Linear quadratic optimal control theory [6] was widely used to derive kinds of guidance laws for tactical missiles in the terminal homing phase. Using the linear quadratic optimal control, the optimal guidance law (OGL) with zero miss distance and impact angle as terminal constraints can be derived [7–14]. Nevertheless, all the guidance laws aforementioned were based on optimal control theory, firstly studied a biased PNG law by adding a time-varying bias term in the conventional PNG to achieve the terminal impact angle [4, 15]. The biased PNG laws with impact angle can also be seen in [16, 17]. Although PNG and its improved forms were widely used in application, there are kinds of drawbacks, such as weak performances against large manoeuvring targets, poor immunity, and low guided precision.

Recently, due to the strong robustness and effectiveness to non-linear systems, the SMC theory [18] was used gradually for designing guidance law with impact angle constraint. Using the SMC theory, the guidance laws with angle constraint were designed by many researchers [19–23]. However, a serious problem of the SMC guidance law is high-frequency chattering of the system. To solve this problem, the robust guidance law was designed through improving the SMC guidance law [24–26]. Compared with the OP and the biased PNG guidance laws, the SMC guidance laws were more efficient in the control of time-varying uncertain systems. However, there was an important problem to address in designing the SMC guidance laws, namely limiting the high-frequency chattering, which is due to the inherent discontinuous switching characteristics of the SMC.

In this paper, we propose a guidance law with impact angle constraint. The main idea of the proposed guidance law is that the non-linear sliding surface is designed to introduce a smooth missile motion along the desired impact angle frame. By using the adaptive neuro-fuzzy inference system (ANFIS), we can reduce the high-frequency chattering of acceleration control command. The ANFSMC guidance law combines the SMC methodology with the ANFIS to enhance the robustness of the system. Although the manoeuvring targets are not considered in the derivation of the method, it also be used to intercept the manoeuvring targets through changing the form of sliding surfaces. Moreover, the chattering problem of the SMC is solved by applying the ANFIS. Therefore, the proposed guidance law, not only has a good robustness with respect to uncertainty, but also hardly arises the chattering to the system.

This paper is organised as follows. In Section 2, we establish the equations of motion for the missile-target engagement and derive the relation between line-of-sight (LOS) angle and impact angle. In Section 3, a guidance law based on the SMC methodology is designed to eliminate the miss-distance and achieve a desired impact angle. We propose the robust SMC guidance law based on the ANFIS to reduce the high-frequency chattering of acceleration command in Section 4. In Section 5, basic performance test and comparison simulation studies are carried out to investigate the feasibility of the ANFSMC guidance law. Finally, conclusions and possible future work are discussed in Section 6.

2 Problem formulation

In this work, planar motion of the missile and target is considered. In order to facilitate the whole analysis of the proposed method, some general assumptions are introduced as follows:

A1. The missile and the target are considered as moving geometric points in a plane.

A2. The missile velocity V_M and heading angle θ_M can be measured by an inertial navigation system (INS).
A3. The target velocity V_T and heading angle θ_T are estimated by the tracking system of the missile.
A4. The velocities of missile V_M and target V_T are constant.
A5. $V_M > V_T$, as the guidance problem with impact angle constraint occurs when the missile has higher manoeuvre capability compared with the target. Here, we denote the target-to-missile velocity ratio, $\nu = V_T/V_M$, which satisfies $\nu < 1$.
A6. The LOS angle and it's rate can be obtained by the missile.

With these assumptions, we assume the velocities of the missiles and the targets are constant. Firstly, we focus on proposing a novel method, which can be used to guide the missiles to intercept the stationary or slowly moving targets. In this method, we can change the form of sliding surface to meet the requirements against manoeuvring targets. Secondly, the constant velocities of the missiles and the targets are feasible in some special applications, such as the cruise missile attack the cave target. At the phase of terminal guidance usually lasting few seconds, we can assume that the velocity of the missile is constant.

2.1 Definition of the impact angle

Consider the planar engagement between the missile and the slowly moving target as shown in Fig. 1, where (x_M, y_M) and (x_T, y_T) denote the position of the missile and the target, respectively. The missile heading angle, velocity, and lateral acceleration are expressed by θ_M, V_M, and A_M, respectively. Similarly, the target heading angle, velocity, and lateral acceleration are expressed by θ_T, V_T, and A_T, respectively. The lateral acceleration command is normal to the velocity vector of the missile. Since only non-manoeuvring targets are considered, the target lateral acceleration $A_T = 0$. q is the LOS angle and R is the relative distance between the missile and the target. η_M and η_T represent the heading error of the missile and the target.

Impact angle control guidance laws aim at achieving interception at a desired impact angle. Here, we define the impact angle, denoted by θ_{imp}, as the angle between the missile velocity vector and the target [25]. The impact angle geometry can be seen in Fig. 2.

Denoting the heading angles of the missile and the target at the impact point as θ_{Mf} and θ_{Tf}, respectively, the impact angle, θ_{imp}, is defined as

$$\theta_{imp} = \theta_{Tf} - \theta_{Mf} \tag{1}$$

Here, for the case of stationary targets, we define the heading angle of the target as $\theta_{Tf} = 0$, and for the slowly moving targets supposed they do not move along the X-axis with a definite

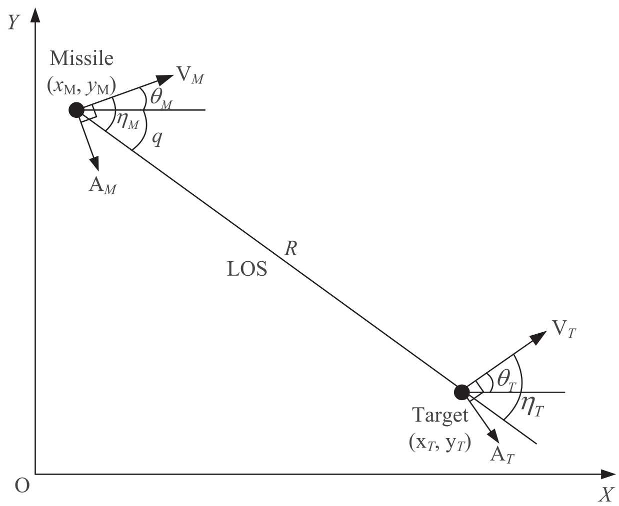

Fig. 1 *Missile/target engagement geometry*

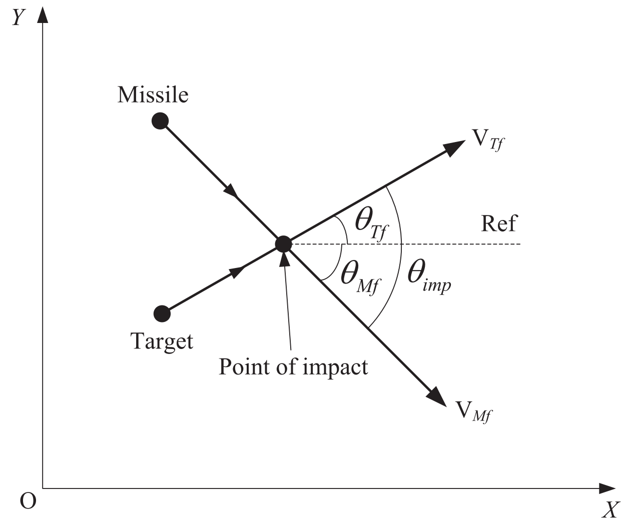

Fig. 2 *Definition of impact angle*

heading angle ($\theta_T \neq 0$), the terminal heading angle of the target satisfies $\theta_{Tf} \neq 0$.

2.2 Equations of motion

The problem is described as follows: the missile should intercept the target at the desired impact angle θ_{imp} for any launch or mid-engagement conditions. To solve this problem, we design a time-varying control value, A_M, to make the missile change the heading angle until intercept the target satisfying the miss distance with respect to impact angle. Under the assumption that velocities of the missiles and the targets are constant, we describe the non-linear kinematics-based engagement dynamics as

$$\dot{R} = V_T \cos\eta_T - V_M \cos\eta_M \tag{2}$$

$$R\dot{q} = V_M \sin\eta_M - V_T \sin\eta_T \tag{3}$$

$$\dot{\theta}_M = A_M/V_M, \quad \dot{\theta}_T = 0 \tag{4}$$

where $\eta_M = q - \theta_M$, and $\eta_T = q - \theta_T$. Here, we assume that the targets are stationary or slowly moving, so the rate of target heading angle is zero. If the targets are manoeuvring, the rate of target heading angle is not equal to zero. The guidance law tries to force the LOS angle rate ($\dot{q}$) to zero. The relation between the impact angle and LOS angle is derived in Section 2.3.

2.3 Relating the impact angle and LOS angle

According to (3), we can derive the corresponding LOS angle, denoted as q_f, as follows

$$V_M \sin\eta_{Mf} = V_T \sin\eta_{Tf} \tag{5}$$

$$\eta_{Mf} = q_f - \theta_{Mf}, \quad \eta_{Tf} = q_f - \theta_{Tf} \tag{6}$$

which from (3), corresponds to when $R\dot{q} = 0$. Thus, we can obtain the LOS angle at the interception course, q_f, which can be given by

$$q_f = \theta_{Tf} - \arctan\left(\frac{\sin\theta_{imp}}{\cos\theta_{imp} - \nu}\right) \tag{7}$$

where q_f and θ_{imp} are the LOS angle and impact angle at the impact time, respectively. On the basis of this relation, we can conclude that there exists a unique relationship between q_f and θ_{imp}. Hence, we can transfer the impact angle constraint to the LOS angle control problem in the design of guidance laws.

3 Non-linear sliding surface design

In fact, the missile is affected by the aerodynamic errors, the measure noise, the time-varying velocity, as well as the target manoeuvring which would seriously worsen the guidance performance. To enhance the robustness of the guidance system, we derive a robust SMC guidance law based on the ANFIS, which is applied to reduce the high-frequency chatting introduced by the inherent discontinuous switching characteristics of the SMC [27].

To obtain the proper lateral acceleration $A_{\rm M}$ in order to make the LOS angle tend to the desired angle, we design a non-linear sliding surface.

Differentiating (3) with respect to t, and according to (2), we can obtain that

$$\begin{aligned}\dot{R}\dot{q} + R\ddot{q} = &-\dot{R}\dot{q} + \dot{V}_{\rm M}\sin(q-\theta_{\rm M}) - V_{\rm M}\dot{\theta}_{\rm M}\cos(q-\theta_{\rm M}) \\ &-\dot{V}_{\rm T}\sin(q-\theta_{\rm T}) + V_{\rm T}\dot{\theta}_{\rm T}\cos(q-\theta_{\rm T})\end{aligned} \tag{8}$$

From (8), we can get the second-order derivatives of the LOS angle, $\ddot{q}$, which can be written as follows

$$\ddot{q} = \frac{-2\dot{R}\dot{q}}{R} - \frac{A_{\rm c}}{R} + \frac{A_{\rm d}}{R} \tag{9}$$

where $A_{\rm c} = V_{\rm M}\dot{\theta}_{\rm M}\cos(q-\theta_{\rm M}) - \dot{V}_{\rm M}\sin(q-\theta_{\rm M})$ and $A_{\rm d} = V_{\rm T}\dot{\theta}_{\rm T}\cos(q-\theta_{\rm T}) - \dot{V}_{\rm T}\sin(q-\theta_{\rm T})$. Due to the assumption that the velocity of the missile is constant, we can obtain that the rate of the missile velocity is zero ($\dot{V}_{\rm M}(t) = 0$). According to (4), $A_{\rm c}$ can be expressed as $A_{\rm c} = A_{\rm M}\cos(q-\theta_{\rm M})$, and it is the missile normal acceleration $A_{\rm M}$ in the vertical projection on the LOS. Hence, we can use $A_{\rm c}$ as control variable in designing guidance law.

Similarly, $A_{\rm d}$ can be expressed as $A_{\rm d} = A_{\rm T}\cos(q-\theta_{\rm T})$. However, it is hard to obtain the $A_{\rm T}$ and $\theta_{\rm T}$. Therefore, we regard $A_{\rm d}$ as the bounded disturbances introduced by the target.

Considering the impact angle constraint, we choose the state variables $x_1 = q(t) - q_f$ and $x_2 = \dot{q}(t)$. Hence, we can get the state equations

$$\begin{bmatrix}\dot{x}_1 \\ \dot{x}_2\end{bmatrix} = \begin{bmatrix} 0 & 1 \\ 0 & -\dfrac{2\dot{R}}{R}\end{bmatrix}\begin{bmatrix}x_1 \\ x_2\end{bmatrix} + \begin{bmatrix}0 \\ -\dfrac{1}{R}\end{bmatrix}A_{\rm c} + \begin{bmatrix}0 \\ \dfrac{1}{R}\end{bmatrix}A_{\rm d}. \tag{10}$$

In this guidance law, the LOS angle rate is required to be zero, which can be written as $\dot{q}_f = 0$, in order to guarantee the missile intercept the target with zero miss distance constraint. To satisfy the terminal angle constraint, we make the LOS angle error to be zero, which can be written as $q(t_f) - q_f = 0$, where t_f represents the terminal time and q_f represents the desired LOS angle. We consider the following switching function

$$s(t) = x_2(t) - \lambda\frac{\dot{R}(t)}{R(t)}x_1(t), \quad R(t) \neq 0 \tag{11}$$

where λ is a positive constant. When $R(t) \geq d_f$ (d_f is a positive constant, noted desired miss distance), we use the proposed guidance law to guide the missile to intercept the target; when $R(t) < d_f$, stop guiding the missile while the missile could rely on inertia hit the target.

To guarantee that the state of system (10) approaches the sliding mode $s = 0$ with a good dynamic performance, a general reaching law is selected as [28]

$$\dot{s}(t) = k\frac{\dot{R}(t)}{R(t)}s(t) - \frac{\varepsilon}{R(t)}{\rm sgn}(s(t)) \tag{12}$$

where k is the positive constants, ${\rm sgn}(s(t))$ is the signum function of $s(t)$ and ε is a proper positive constant and the value of it may introduce great influence on the system, noted as a reaching law coefficient. The value of ε may bring great influence to the system, if the value is too small, the convergence of the system will become slow, if the value is too large, the system may cause chattering easily. When the distance $R(t)$ between the missile and the target is large, the reaching law coefficient ε should be properly lower its value in order to make the control acceleration no more than the peak normal load; when $R(t)$ is small, ε should be rapidly larger its value to suppress divergence of LOS angle rate $\dot{q}$ so that guarantee the intercept accuracy. Though the value of ε is relevant to the distance between the missile and the target, it can be chose by a proper constant through humans knowledge.

The goal is to find the time-varying control acceleration $A_{\rm c}$ that the sliding surface (11) and its derivative (12) will both converge to zero in finite time. To analyse the stability of the proposed sliding surface, we choose a Lyapunov function V as

$$V = \frac{1}{2}s^2 \tag{13}$$

Taking the time derivative of (13) leads to

$$\dot{V} = s\dot{s} \tag{14}$$

Substituting (12) into (14) gives

$$\dot{V} = k\frac{\dot{R}(t)}{R(t)}s^2(t) - \frac{\varepsilon}{R(t)}|s(t)| \tag{15}$$

when $s(t) \neq 0$, because of $\dot{R}(t) < 0$, we can derive $\dot{V} < 0$. Hence, the state of the system will converge to the sliding surface $s = 0$. Therefore, the system have a good performance of stability.

According to (10) and (12), and differentiating (11), we can obtain the SMC guidance law

$$A_{\rm c} = -(2+\lambda+k)\dot{R}x_2 + \lambda(k+1)\frac{\dot{R}}{R}x_1 + \varepsilon\,{\rm sgn}s + A_{\rm d} \tag{16}$$

where k and λ are the appropriately chosen positive constants, and $A_{\rm d}$ is the disturbance introduced by the uncertainty of the target.

Since the variable $A_{\rm d}$ is regarded as the disturbance, we can simplify the SMC guidance law as follows

$$A_{\rm c} = -(2+\lambda+k)\dot{R}x_2 + \lambda(k+1)\frac{\dot{R}}{R}x_1 + \varepsilon\,{\rm sgn}s. \tag{17}$$

In the above guidance law (17), there is a signum function variable $\varepsilon\,{\rm sgn}s$, which may introduce the high-frequency chattering of the system. To reduce the high-frequency chatting, we use the ANFIS to solve this problem.

4 Robust guidance law design

Although the sliding mode correction term is utilised to improve the robustness of the guidance system, the high-frequency chattering would instead decrease the control precision in practical application. From (17), it can be found that the performance of the robust guidance laws mainly lies on the signum function item, $\varepsilon\,{\rm sgn}s$. Usually, the methods of reducing the high-frequency chattering are using the high gain continuous function or a saturation function to replace the signum function. However, it is difficult to choose a proper value which can meet the robustness of the guidance laws and reduce the high-frequency chattering in designing the guidance laws against manoeuvring targets. Since the ANFIS has the self-learning ability, we use it to reduce the high-frequency chattering and enhance the robustness of the system.

In this work, the ANFIS is applied to adaptive update the additional lateral acceleration (latax) command since it has a powerful self-learning ability. As shown in Fig. 3, the ANFIS is a five-layer network in substance. For this inference system, the inputs are $s(t)$ and $\dot{s}(t)$ and the output is the additional control acceleration $A_{\rm c2}$ to

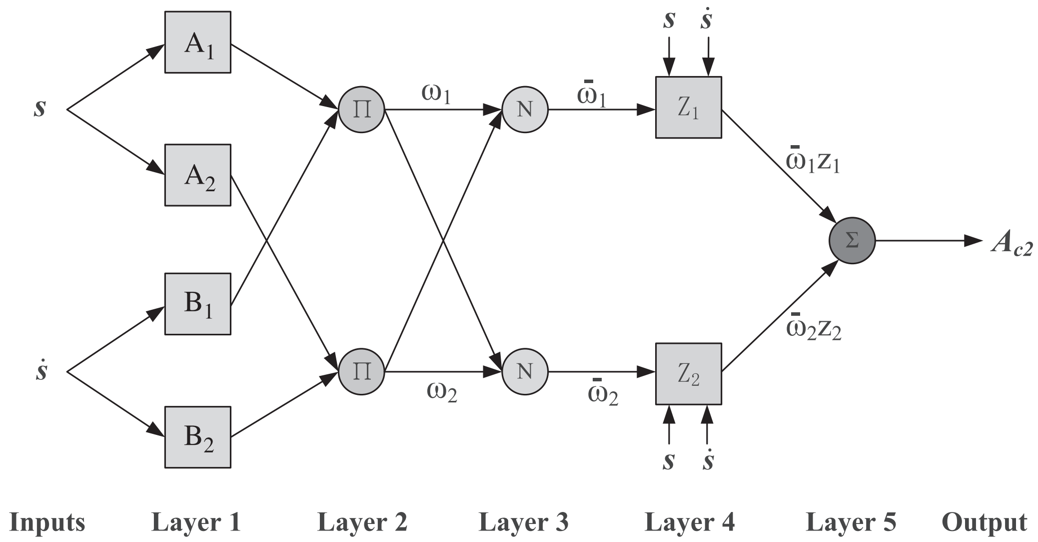

Fig. 3 *Schematic diagram of ANFIS*

enhance the robustness of the system. Hence, the robust guidance law with impact angle constraint can be written as

$$A_c = A_{c1} + A_{c2} \tag{18}$$

where $A_{c1} = -(2 + \lambda + k)\dot{R}x_2 + \lambda(k + 1)(\dot{R}/R)x_1$.

4.1 ANFIS design

Jang [29] proposed the ANFIS which is a fuzzy inference system based on Takagi–Sugeno model. The ANFIS structure [30] is similar to the neural network as shown in Fig. 3. Considering the neural network's physical meaning is not clear and the learning rate is slow, we use the ANFIS in designing guidance law. The ANFIS uses fuzzy reasoning similar to human reasoning, and has the fast learning ability.

In this inference system, the inputs are $s(t)$ and $\dot{s}(t)$ and the output is the additional control acceleration A_{c2}. There are various ANFIS architectures, but the one using a first-order Sugeno fuzzy model is the most common [31]. The detailed Sugeno fuzzy model was presented in [32]. Using one-order Sugeno fuzzy model, we can suppose two fuzzy if–then rules as follows

(i) If $s(t)$ is A_1, and $\dot{s}(t)$ is B_1, then $A_{c21} = p_1 s(t) + q_1\dot{s}(t) + r_1$
(ii) If $s(t)$ is A_2, and $\dot{s}(t)$ is B_2, then $A_{c22} = p_2 s(t) + q_2\dot{s}(t) + r_2$

where $A(i)$ and $B(i)$ $(i = 1, 2)$ are the fuzzy sets with respond to two inputs $(s, \dot{s})$.

Suppose the membership functions of inputs $A(i)$ and $B(i)$ are Sugeno model, which can be written as

$$s_{Ai}(s, a_i, b_i) = \frac{1}{1 + e^{-a_i(s-b_i)}} \tag{19}$$

$$s_{Bi}(\dot{s}, c_i, d_i) = \frac{1}{1 + e^{-c_i(\dot{s}-d_i)}} \tag{20}$$

where $\{a_i, b_i\}$ and $\{c_i, d_i\}$ $(i = 1, 2)$ are the characteristic parameters of their Sugeno functions.

Hence, if the characteristic parameters are changed, their Sugeno functions are altered and the membership functions will be adaptively altered.

According to the above interference system, we can make the five-layer network. The node functions in the same layer are of the same function family as described below:

Layer 1: The node in this layer is square node with a node function

$$O_{1,i} = u_{Ai}(s), \quad i = 1, 2 \tag{21}$$

$$O_{1,i} = u_{Bi}(\dot{s}), \quad i = 3, 4 \tag{22}$$

where s (or $\dot{s}$) is the ith node input, A (or B) is linguistic label (small or large) associated with this node function. In other words, $O_{1,i}$ is the membership function of A and B, and it specifies the degree to which the given s satisfies the quantifier A_i $(i = 1, 2)$, and $\dot{s}$ satisfies the quantifier B_i $(i = 3, 4)$.

Layer 2: Every node in this layer is a circle node labelled $\prod$ which multiplies the incoming signals and sends the product out, which can be written as follows

$$O_{2,i} = \omega_i = u_{Ai}(s) \times u_{Bi}(\dot{s}), \quad i = 1, 2 \tag{23}$$

Each node output represents the firing strength of a rule.

Layer 3: Every node in this layer is a circle node labelled N which calculates the ratio of the related incentive strength to the total, which is showed as follows.

$$O_{3,i} = \bar{\omega}_i = \frac{\omega_i}{\omega_1 + \omega_2} \tag{24}$$

For convenience, outputs of this layer is called normalised firing strengths.

Layer 4: Each node i in this layer is a square node with a node function

$$O_{4,i} = \bar{\omega}_i z_i = \bar{\omega}_i(p_i s + q_i\dot{s} + r_i) \tag{25}$$

where $\bar{\omega}_i$ is the normalisation of incentive strength from layer 3, and $\{p_i s + q_i\dot{s} + r_i\}$ is the parameter set. Parameters in this layer are referred to the consequent parameters.

Layer 5: The single node in this layer is a circle node labelled $\sum$ that calculates the overall output as the summation of all incoming signals, which can be calculated as follows

$$O_{5,1} = \sum_i \bar{\omega}_i z_i = \frac{\sum_i \omega_i z_i}{\sum_i \omega_i} \tag{26}$$

In this way, an adaptive network based on the Sugeno fuzzy models is constructed.

To make the guidance system have a faster response time, we train the ANFIS structure offline by saturation function, substituting signum function, and update the additional control demand adaptively in time. The saturation function can be written as

$$\text{sat}(s) = \frac{s}{|s| + \sigma} \tag{27}$$

The ANFIS combines the fuzzy control with the neural network control, so it has good performances in utilising the expert knowledge to draw the inference rules and has self-learning and self-adaptive abilities.

4.2 Reaching law design

In this section, a novel robust guidance law, denoted the adaptive neuro-fuzzy SMC (ANFSMC) guidance law, is designed. The proposed control system include the SMC and the ANFIS. The structure of the ANFSMC guidance law is shown in Fig. 4.

Considering the peak normal load of the missile, we establish the latax bound block, as shown in Fig. 4. In this study, the latax $A_{\rm M}$ is bounded according to the saturation function

$$A_{\rm M} = \begin{cases} A_{\rm Mmax}\text{sgn}(A_{\rm c}), & \text{if } |A_{\rm c}| \ge A_{\rm Mmax} \\ A_{\rm c}, & \text{if } |A_{\rm c}| < A_{\rm Mmax} \end{cases} \tag{28}$$

where $A_{\rm Mmax}$ is the latax bound imposed on the missile. In (28), we use the signum function variable $A_{\rm Mmax}\text{sgn}(A_{\rm c})$ to restraint the value of $A_{\rm M}$. It will not introduce the high-frequency chattering, but it can make the proposed guidance law meet the overloading constraint.

At the engagement course, we can stop the guidance to the missile if the distance between the missile and the target is less than defined miss distance. The error distance can be written as

$$e_{\rm d}(t) = d_f - R(t) \tag{29}$$

where d_f is the defined miss distance, and $R(t)$ is the distance between the missile and the target in real time. Hence, if $e_{\rm d}(t) \ge 0$, stop the guidance that the missile will intercept the target relying on inertia; if $e_{\rm d}(t) < 0$, then we should continue the guidance based on the ANFSMC guidance law.

4.3 Comparison guidance laws

To demonstrate the performance and characteristics of the proposed guidance law, three other different guidance laws – OP guidance law proposed in [13], the SMC guidance law as in (17), and the SMC guidance law based on the BP neural network – are introduced.

4.3.1 OP guidance law: Consider the following optimal control problem: find a proper acceleration command $A(t)$ which minimises

$$J_1 = \frac{1}{2}\int_{t_0}^{t_f} A(s)^2\,\text{d}s \tag{30}$$

where t_0 and t_f are the initial and terminal times, respectively. A is the acceleration command normal to the velocity vector to change the missile heading angle.

The solution of this linear quadratic optimal control problem, known as the biased PNG in [33], can be derived as follows

$$A_{\rm OP} = \frac{6y_{\rm go}}{x_{\rm go}^2} - \frac{4\theta_{\rm M}}{x_{\rm go}} - \frac{2\theta_{{\rm M}f}}{x_{\rm go}} \tag{31}$$

where $y_{\rm go} = y_f - y$, and $x_{\rm go} = x_f - x$. $\theta_{{\rm M}f}$ is the desired terminal angle of the missile. Note that this solution can control the missile to intercept the target with impact angle.

4.3.2 SMC guidance law: The SMC guidance law is derived in this paper. The control command can be given by (17). In this guidance law, there is a signum function which may cause the control command chattering. Ignoring the chattering of the system, the SMC guidance law can satisfy the impact angle and zero miss distance constraints.

4.3.3 SMC guidance law based on the BP neural network: Due to the excellent ability of non-linear mapping, self-organisation, and self-learning, artificial neural networks (ANNs) have proven to be of widespread utility in engineering [34]. In this paper, we combine the BP neural network with the SMC to obtain the latax control command.

The input and output data of the BP neural network are similar to the ANFIS. Hence, the inputs are $s(t)$ and $\dot{s}(t)$ and the output is the additional control acceleration $A_{\rm c2BP}$. The BP neural network is a typical feed-forward neural network. There are three layers of BP neural network, which are input layer, hidden layer and output layer. Applied this neural network, we can predict the additional control command $A_{\rm c2BP}$ according to inputs values of $s(t)$ and $\dot{s}(t)$ in real time. Hence, the SMC guidance law based on the BP neural network (BPSMC) can be expressed as

$$A_{\rm BPSMC} = A_{\rm c1} + A_{\rm c2BP} \tag{32}$$

where $A_{\rm c1}$ has the same expression with the SMC guidance law in (18), and $A_{\rm c2BP}$ is the additional control acceleration.

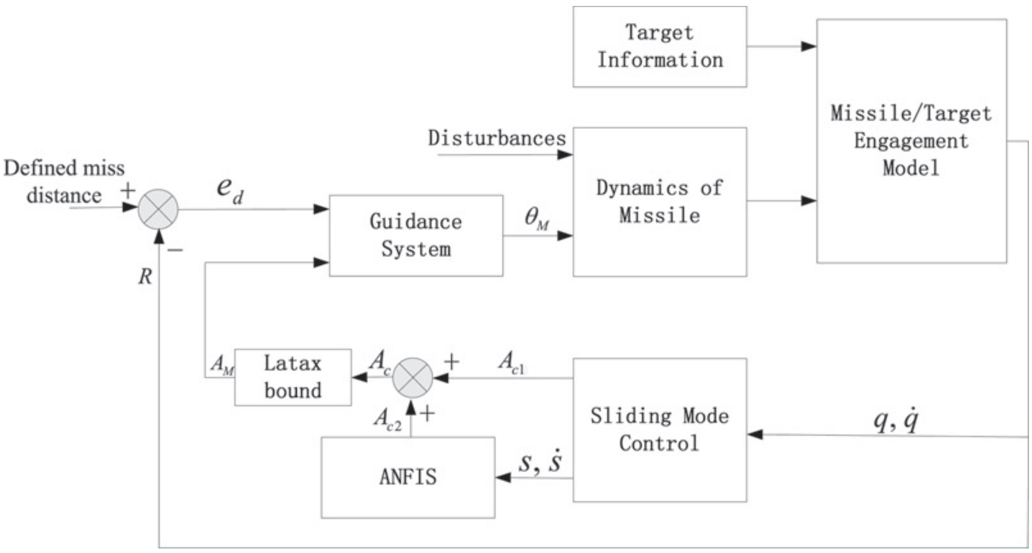

Fig. 4 *Robust guidance law structure*

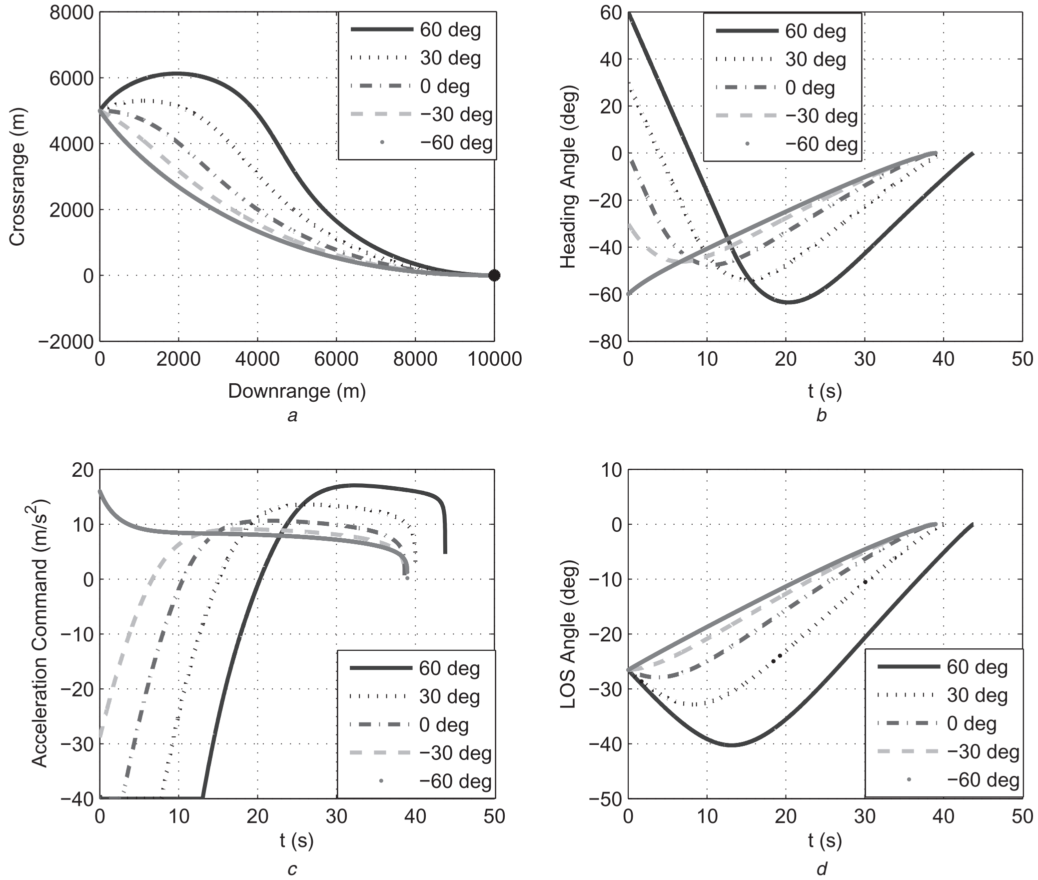

Fig. 5 *Results with various initial heading angles*

a Missile–target engagement
b Missile heading angle
c Missile acceleration command
d Missile LOS angle

5 Simulation results

To demonstrate and evaluate the performances of proposed guidance law, some simulation results are provided in this section. Firstly, we present the simulation for showing the ability of impact angle control with various initial heading angles. Moreover then we compare the ANFSMC guidance law with other guidance laws, such as the OP guidance law, the SMC guidance law, and the BP network SMC guidance law. It is assumed that the missile has perfect measurements on θ_M, q, and R, and there are command limitations with latax bound as (28).

5.1 Basic performance test of the ANFSMC guidance law

The homing geometry as described in Fig. 1 with following parameters are used to setup the test engagement scenario. Firstly, we outline parameters for homing geometry. The initial position of the missile is $(0, 5000)$ m, and the velocity is 300 m/s. A stationary target is placed on $(10,000, 0)$ m. Moreover then the ANFSMC guidance law parameters are given as follows. The SMC guidance law parameters are defined as follows: $k = 1.8$, $\lambda = 1.5$, and $\varepsilon = 50$. The saturation function parameter is defined as $\sigma = 0.01$. The latax bound is chosen as $A_{\text{Mmax}} = 40\,\text{m/s}^2$. The defined error distance is assumed to be $d_f = 5$ m. In the design of the ANFIS, the membership functions of inputs are both bell-shaped which can be expressed by *gbellmf* in MATLAB, and output is *linear*. The training numbers of the ANFIS are 100, which can be written as *numEpochs* = 100.

In the first engagement scenario studied, the missile initial heading angles are different (60°, 30°, 0°, −30°, and −60°), and the impact angle is 0°. The results for this simulation are represented in Fig. 5. From Fig. 5, it can be clearly seen that the ANFSMC guidance law leads to a desired angle of 0° with various initial heading angles of the missile. The acceleration command profiles in Fig. 5*c* show a good performance in latax bound. The missile heading angle and LOS angle are shown in Figs. 5*b* and *d*. The detailed simulation results are given in Table 1. At the final time, the missile LOS angle and heading angle are converged to zero as the missile approaches the target so that the desired impact angle is successfully achieved. By introducing the latax bound, smooth acceleration commands are also be obtained and the miss distance at the final time is smaller than 5 m in simulation cases.

5.2 Performance comparison

To compare the robust performance of the ANFSMC guidance law with other guidance laws (OP, SMC, and BPSMC), we conduct the following simulation experiments.

Table 1 Detailed results with various initial heading angles

Initial heading angle, deg.	Miss distance, m	Impact angle error, deg.	Maximum latax command, m/s²	Flight time, s
60	4.76	0	−40.00	43.78
30	4.70	0	−40.00	40.01
0	3.64	0	−40.00	38.63
−30	4.97	0	−28.89	38.57
−60	3.55	0	16.01	38.99

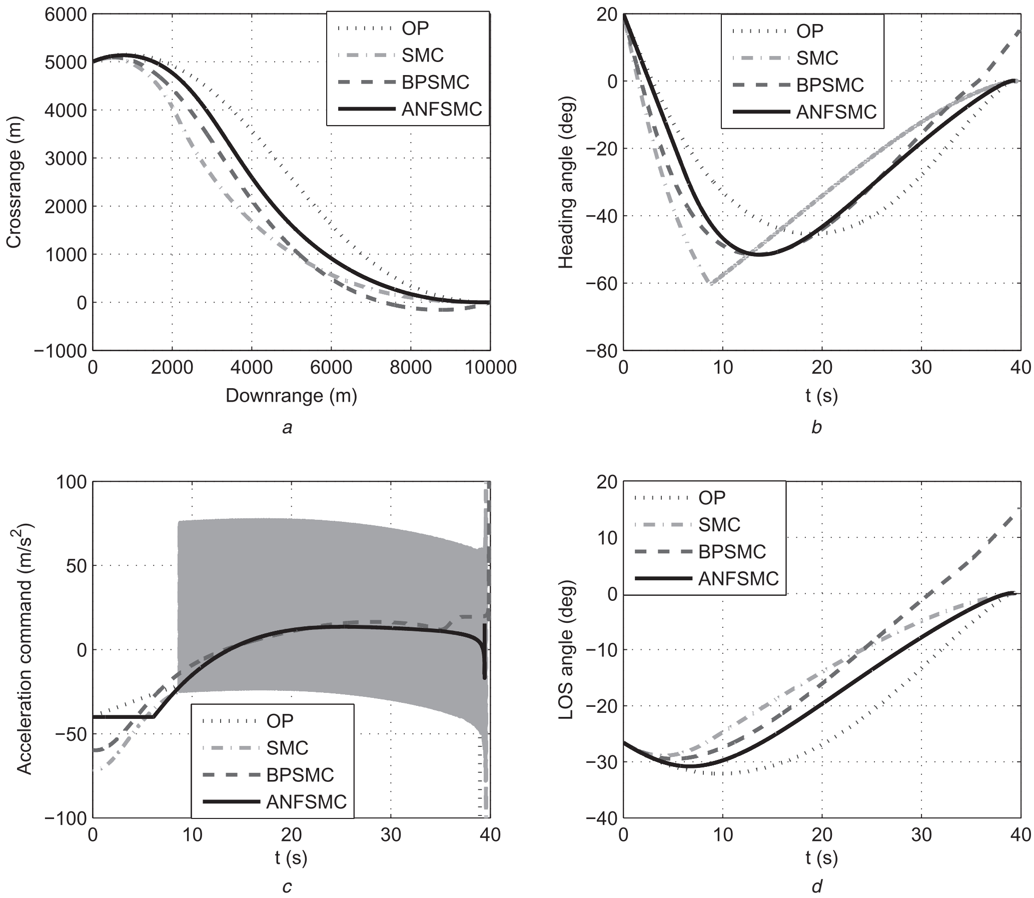

Fig. 6 *Performance comparison against stationary target*

a Missile–target engagement
b Missile heading angle
c Missile acceleration command
d Missile LOS angle

5.2.1 Stationary target: In this section, the results for the case of stationary targets are presented. The initial positions of the missile and the target are used in the previous simulations. The impact angle is defined as $\theta_{\text{imp}} = 0°$ to achieve lateral attack against the stationary target. We consider the missile is guided using the OP guidance law, the SMC guidance law, the BPSMC guidance law, and the ANFSMC guidance law. The results for this simulation are represented in Fig. 6.

From Fig. 6*a*, we deduce that the missile could approach the stationary target through applying different guidance laws. However, the acceleration command is significantly different among these guidance laws, which can be seen in Fig. 6*c*. The acceleration command of the SMC guidance law present the high-frequency chattering. What's more, the acceleration command of the OP, SMC, and BPSMC guidance laws excess the maximum allowable latax A_{Mmax} at some times. In this aspect, the ANFSMC guidance law have a good performance for $|A_{\text{M}}| \leq A_{\text{Mmax}}$. From Figs. 6*b* and *d*, it can be seen that the OP, ANFSMC, and SMC guidance law guide the missile to successively approach the target with impact angle much before interception actually occurs, but the BPSMC guidance law has a larger error in impact angle. The detailed results are shown in Table 2. The missile can intercept the stationary target by using these guidance laws with miss distance is smaller than 5 m in finite time. The OP, SMC, and ANFSMC guidance laws satisfy the impact angle constraint with impact angle error is smaller than 3°. However, only the ANFSMC guidance law meet the latax bound constraint. Hence, the ANFSMC guidance law present better performances than other guidance laws with impact angle constraint against stationary target.

5.2.2 Slowly moving target: The proposed guidance law can also be applied to engage moving targets. In order to demonstrate the effectiveness of the ANFSMC guidance law, a simulation is performed for a moving target. The initial positions from in the previous simulations are used here too, only now the target is assumed to be moving at a velocity of $V_{\text{T}} = 60\,\text{m/s}$ with a heading angle of $\theta_{\text{T}} = 20°$. The target velocity is smaller than the missile velocity, so it satisfies the assumption of $\nu = \frac{V_{\text{T}}}{V_{\text{M}}} < 1$. For slowly moving target tests, we consider the desired impact angle $\theta_{\text{imp}} = 0°$. The missile terminal heading angle should reach the specified angle $\theta_{\text{M}f} = \theta_{\text{T}f} = 20°$ and the terminal LOS angle should be $q_f = 20°$.

Fig. 7 presents the comparison results against slowly moving target: missile-target engagement trajectories, missile heading angle, missile acceleration command, and missile LOS angle. As shown in Fig. 7*a*, using various guidance laws, the missile can approach the slowly moving target with the desired angle in finite time. From Figs. 7*b* and *d*, we can observe that the OP, SMC, ANFSMC guidance laws can guide the missile to intercept the slowly

Table 2 Detailed results against stationary target

Guidance law	Miss distance, m	Impact angle error, deg.	Maximum latax command, m/s^2	Flight time, s
OP	0.75	0.16	−558.84	39.00
SMC	1.8	0.34	−809.57	39.59
BPSMC	1.69	15.22	289.61	39.87
ANFSMC	1.64	0.03	−40.00	39.44

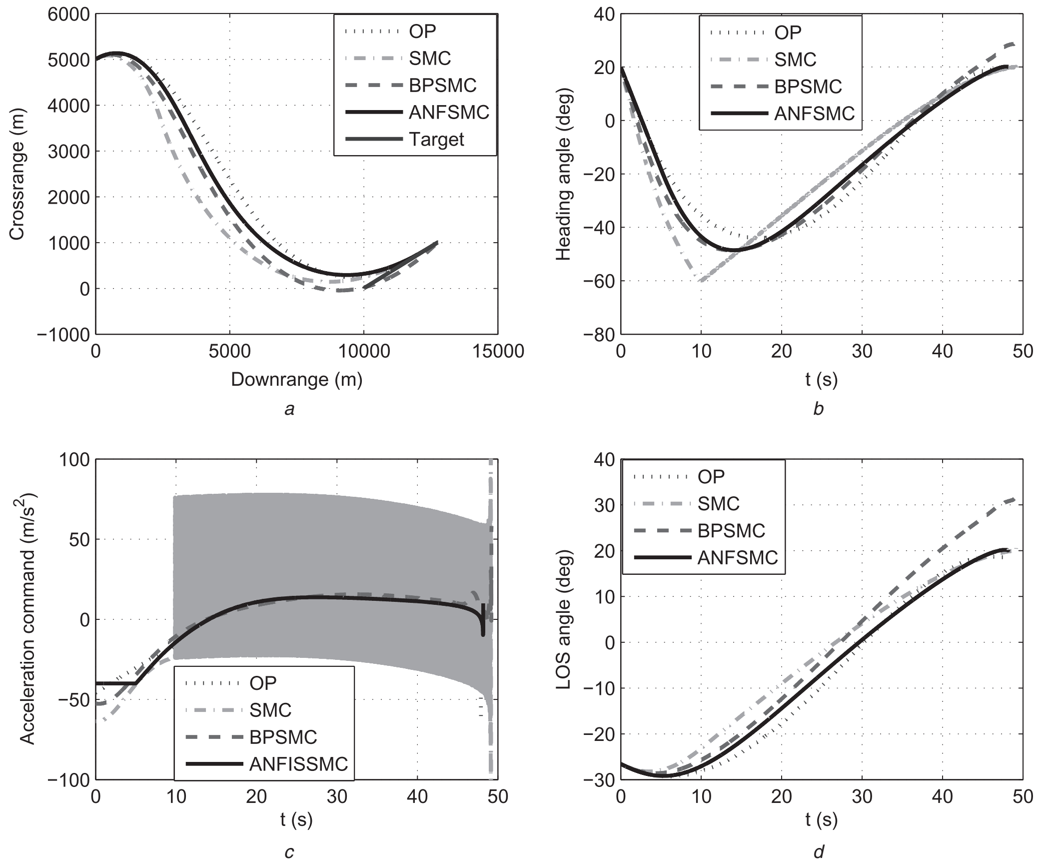

Fig. 7 *Performance comparison against slowly moving target*

a Missile–target engagement
b Missile heading angle
c Missile acceleration command
d Missile LOS angle

Table 3 Detailed results against slowly moving target

Guidance law	Miss distance, m	Impact angle error, deg.	Maximum latax command, m/s^2	Flight time, s
OP	3.38	1.16	−61.36	47.90
SMC	0.72	0.49	−479.98	49.15
BPSMC	2.53	9.49	58.17	49.21
ANFSMC	2.50	0.03	−40.00	48.15

moving target with small desired heading angle errors and LOS angle errors, but the BPSMC guidance law has larger errors in desired heading angle and LOS angle. From Fig. 7*c*, we can see that the missile acceleration command of the ANFSMC guidance law satisfies the latax bound. However, the missile acceleration commands of the OP, SMC, and BPSMC guidance laws are not within the latax bound at some time. What's more, the missile acceleration command of the SMC guidance law occur high-frequency chattering which would lead to system instability. The detailed comparison results are shown in Table 3. From Table 3, we can see that only the ANFSMC guidance law meets the multiple constraints, such as miss distance, impact angle, and latax bound. Hence, we can draw the conclusion that the performances of the ANFSMC guidance law are superior to the OP, SMC, BPSMC guidance laws in terms of latax command, miss distance, and impact angle error.

6 Conclusions

In this paper, the ANFSMC guidance law is proposed for the missile intercept the target with impact angle constraint. It is characterised as the combination of the SMC and the ANFIS, which has a good performance on limiting the high-frequency chattering of acceleration command. Some numerical simulations show that the proposed guidance law is capable of interception at the desired impact angle and latax bound constraints. Moreover, the proposed guidance law provides out performances and feasibility with impact angle than the OP, SMC, and BPSMC guidance laws against stationary and slow moving target.

In future work, we should consider several factors such as the autopilot lag or aerodynamic model of the missile for the practical implementation. What's more, the ANFSMC guidance law could be extended to the general case of a high manoeuvring target. Although the manoeuvring target is not considered in the derivation of the theory, the proposed guidance law can also be utilised to intercept the manoeuvring target with impact angle constraint.

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