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Robust observers for neutral jumping systems with uncertain information

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Abstract

In this paper, the problem of designing observer for a class of uncertain neutral systems. The uncertainties are parametric and norm-bounded. Both robust observation and robust $\mathscr{H}_\infty$ observation methods are developed by using linear state-delayed observers. In case of robust observation, sufficient conditions are established for asymptotic stability of the system, which is independent of time delay. The results are then extended to robust $\mathscr{H}_\infty$ observation which renders the augmented system asymptotically stable independent of delay with a guaranteed performance measure. Furthermore, a memoryless state-estimate feedback is designed to stabilize the closed-loop neutral

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system. In all cases, the gain matrices are determined by linear matrix inequality approach. Two numerical examples are presented to illustrate the validity of the theoretical results.

Keywords: Time-delay systems; Linear neutral systems; Robust observation; Stochastic stability; State-delayed observers; Robust $\mathscr{H}_\infty$ observation

1. Introduction

As is well known, dynamical models of many physical and engineering systems incorporate time-delay factors, see for example, [11,13]. When using these models in the analysis and control design, there have been three basic approaches [1,20]: (1) using infinite-dimensional systems theory by embedding the class of time-delay systems into a larger class of dynamical systems for which the state evolution is described by appropriate operators in infinite-dimensional spaces; (2) applying algebraic systems theory in which the evolution of delay-differential systems is provided in terms of linear systems over rings and (3) employing functional differential systems (FDS) by incorporating the influence of the hereditary effects of system dynamics on the change rate of the system and in this regards, it provides an appropriate mathematical structure. On the other hand, robust observation (or estimation) [5,26,9,28,30] is concerned with the state reconstruction when the plant model has uncertain parameters and is described by ordinary differential equations (ODE) or equivalent representation. Following the third approach based on FDS, results on estimating the state of uncertain system with state-delay are developed in [18] and related work can be found in [21]. An integral part of FDS [11–13,32] is the class of neutral-type systems which can be found in several applications including, but not limited to, chemical reactor, rolling mill, infeed grinding, lossless transmission lines and hydraulic systems. Stability analysis and feedback stabilization for neutral FDS have been studied in [16,27] and other related work was reported in [20]. Recently $\mathscr{H}_\infty$ control has been developed in [19] for a class of linear neutral systems with parametric uncertainties.

On another research front line, there are a wide class of systems having a variable structure subject to random changes which may result from abrupt phenomena such as component and interconnection failures, parameters shifting, tracking and/or variation in the time frame of measurements. Systems with this character may be modelled as hybrid ones; that is, the state space of the system contains both discrete and continuous states and is frequently termed *jumping systems*. Results on the stability and control of linear jumping systems can be found in [3,7,25,14,33,4,2,31,23,24] and the references therein. This paper build upon [20,19] and extends their results further by considering the state observation and stabilization problems for a class of linear neutral systems with norm-bounded uncertainties. Note that the motivation to investigate this kind of system is that some control systems depend not only on state delays but also on derivatives of delayed states, and this class of systems is referred as neutral delay systems. The system we studied in this paper is an extension of the standard neutral systems with jumps. Initially, we address both problems of robust state observation and robust $\mathscr{H}_\infty$ observation and employ a new linear state-delayed observer such that the asymptotic stability of the combined neutral system and the proposed observer is guaranteed for all admissible uncertainties. The main tool for solving the foregoing problems is the linear matrix inequality (LMI) approach. In this regard, it has been established that the solution of robust is expressed in terms of two LMIs involving scaling parameters. Looked at in this light, the developed methods provide new results which in some sense are the dual of [19]. Then, the robust stabilization problem is considered by designing memoryless state-estimate feedback such the asymptotic stability of the closed-loop stability is guaranteed. The analytical developments of this work are organized into theorems whereby the results are presented in a systematic and gradual build-up. Finally, two numerical examples are presented to illustrate the validity of the theoretical results. It should be mentioned that in fact, our paper deals with a class of neutral systems with uncertainties and jumping parameters using the difference operator approach. The method of analysis is systematic and the developed results are new and delay-independent. There is a wide-class systems, such as fault-tolerant systems, satisfying these features.

1.1. Notations and facts

The notation in this paper is fairly standard. We use W^t, W^{-1}, $\lambda(W)$ and $\|W\|$ to denote, respectively, the transpose, the inverse, the eigenvalues and the induced norm of any square matrix W. We use $W > 0$ ($\geqslant$, $<$, $\leqslant 0$) to denote a symmetric positive definite (positive semidefinite, negative, negative semidefinite) matrix W with $\lambda_{\rm m}(W)$ and $\lambda_{\rm M}(W)$ being the minimum and maximum eigenvalues of W and I denote the $n \times n$ identity matrix. The open left half of the complex plane is represented by $\mathbb{C}^-$. The Lebesgue space $\mathscr{L}_2[0,\infty)$ consists of square-integrable functions on the interval $[0,\infty)$. Let $\mathbb{C}_{n,\tau} = \mathbb{C}([-\tau,0],\mathbb{R}^n)$ denote the Banach space of continuous vector functions mapping the interval $[-\tau,0]$ into $\mathbb{R}^n$ with the topology of uniform convergence and equipped with the norm $\|x\|_* \triangleq \sup_{-\tau\leqslant\theta\leqslant 0}\|x\|$ where $\|\cdot\|$ is the Euclidean norm and $\tau > 0$ is termed the *delay factor*. Sometimes, the arguments of a function will be omitted in the analysis when no confusion can arise.

Fact 1 (*Schur Complement*). Given constant matrices Ω_1, Ω_2, Ω_3 where $0 < \Omega_1 = \Omega_1^{\rm t}$ and $0 < \Omega_2 = \Omega_2^{\rm t}$ then $\Omega_1 + \Omega_3^{\rm t}\Omega_2^{-1}\Omega_3 < 0$ if and only if

$$\begin{bmatrix} \Omega_1 & \Omega_3^{\rm t} \\ \Omega_3 & -\Omega_2 \end{bmatrix} < 0 \quad \text{or} \quad \begin{bmatrix} -\Omega_2 & \Omega_3 \\ \Omega_3^{\rm t} & \Omega_1 \end{bmatrix} < 0.$$

Fact 2. Given any real matrices Σ_1, Σ_2, Σ_3 with appropriate dimensions, such that $0 < \Sigma_3 = \Sigma_3^{\rm t}$ the following inequality holds:

$$\Sigma_1^{\rm t}\Sigma_2 + \Sigma_2^{\rm t}\Sigma_1 \leqslant \Sigma_1^{\rm t}\Sigma_3\Sigma_1 + \Sigma_2^{\rm t}\Sigma_3^{-1}\Sigma_2.$$

Fact 3. Let Σ_1, Σ_2, Σ_3 and $0 < R = R^{\rm t}$ be real constant matrices of compatible dimensions and $H(t)$ be a real matrix function satisfying $H^{\rm t}(t)H(t) \leqslant I$. Then for any $\rho > 0$ satisfying $\rho\Sigma_2^{\rm t}\Sigma_2 < R$, the following matrix inequality holds:

$$(\Sigma_3 + \Sigma_1 H(t)\Sigma_2)R^{-1}(\Sigma_3^{\rm t} + \Sigma_2^{\rm t}H^{\rm t}(t)\Sigma_1^{\rm t}) \leqslant \rho^{-1}\Sigma_1\Sigma_1^{\rm t} + \Sigma_3\left(R - \rho\Sigma_2^{\rm t}\Sigma_2\right)^{-1}\Sigma_3^{\rm t}.$$

2. Class of neutral jumping systems

2.1. Model description

Given a probability space $(\Omega, \mathscr{F}, \mathbf{P})$ where Ω is the sample space, $\mathscr{F}$ is the algebra of events and $\mathbf{P}$ is the probability measure defined on $\mathscr{F}$. Let the random process $\{\eta_t, t \in [0, \mathscr{T}]\}$ be a homogeneous, finite-state Markovian process with right continuous trajectories and taking values in a finite set $\mathscr{S} = \{1, 2, \ldots, s\}$ with generator $\Im = (p_{ij})$, $i, j \in \mathscr{S}$ with transition probability from mode i at time t to mode j at time $t + \delta$, $i, j \in \mathscr{S}$:

$$p_{ij} = Pr(\eta_{t+\delta} = j | \eta_t = i) = \begin{cases} \alpha_{ij}\delta + {\rm o}(\delta), & \text{if } i \neq j, \\ 1 + \alpha_{ij}\delta + {\rm o}(\delta), & \text{if } i = j \end{cases} \tag{2.1}$$

with transition probability rates $\alpha_{ij} \geqslant 0$ for $i, j \in \mathscr{S}$, $i \neq j$ and

$$\alpha_{ii} = -\sum_{m=1, m\neq i}^{s} \alpha_{im}, \tag{2.2}$$

where $\delta > 0$ and $\lim_{\delta\downarrow 0}{\rm o}(\delta)/\delta = 0$. The set $\mathscr{S}$ comprises the various operational modes of the system under study.

We consider a class of stochastic uncertain neutral systems with Markovian jump parameters described over the space $(\Omega, \mathscr{F}, \mathbf{P})$ by

$$
\begin{aligned}
(\Sigma_{\Delta n})\quad \mathscr{M}(\dot{x}_t) &\triangleq \dot{x}(t) - B(\eta_t)\dot{x}(t-\tau) \\
&= [A_o(\eta_t) + \Delta A_o(t,\eta_t)]x(t) + [A_d(\eta_t) + \Delta A_d(t,\eta_t)]x(t-\tau) + N(\eta_t)w(t) \\
&= A_{\Delta o}(t,\eta_t)x(t) + A_{\Delta d}(t,\eta_t)x(t-\tau) + N(\eta_t)w(t), \qquad (2.3)
\end{aligned}
$$

$$
x(\eta) = \phi(\eta) \in \mathbb{C}([-\tau,0],\mathbb{R}^n),\quad \forall \eta \in [-\tau,0], \qquad (2.4)
$$

$$
\begin{aligned}
y(t) &= [C_o(\eta_t) + \Delta C_o(t,\eta_t)]x(t) + [C_d(\eta_t) + \Delta C_d(t,\eta_t)]x(t-\tau) + M(\eta_t)w(t) \\
&= C_{\Delta o}(t,\eta_t)x(t) + C_{\Delta d}(t,\eta_t)x(t-\tau) + M(\eta_t)w(t), \qquad (2.5)
\end{aligned}
$$

$$
z(t) = L(\eta_t)x(t), \qquad (2.6)
$$

where $x(t) \in \mathbb{R}^n$ is the system state, $y(t) \in \mathbb{R}^m$ is the measurement output, $z(t) \in \mathbb{R}^p$ is the controlled output, $w(t) \in \mathscr{L}_2[0,\infty)$ is the disturbance input, $z(t) \in \mathbb{R}^r$ is the controlled output which belongs to $\mathscr{L}_2[(\Omega,\mathscr{F},\mathbf{P}),[0,\infty)]$ and the factor $\tau > 0$ is a constant scalar representing the amount of time-lag in the state. Frequently the term $\mathscr{M}(x_t) : \mathbb{C}[-\tau,0] \to \mathbb{R}^n \triangleq x(t) - Bx(t-\tau)$ is called the *difference operator* and it offers a fundamental role in the analytical development throughout the paper.

For each possible value $\eta_t = i$, $i \in \mathscr{S}$, we will denote the system matrices of $(\Sigma_{\Delta n})$ associated with mode i by

$$
\begin{aligned}
&A_o(\eta_t) \triangleq A_o(i),\quad A_d(\eta_t) \triangleq A_d(i),\quad N(\eta_t) \triangleq N(i), \\
&C_o(\eta_t) \triangleq C_o(i),\quad C_d(\eta_t) \triangleq C_d(i),\quad M(\eta_t) \triangleq M(i),\quad B(\eta_t) \triangleq B(i),
\end{aligned} \qquad (2.7)
$$

where $A_o(i) \in \mathbb{R}^{n\times n}$, $A_d(i) \in \mathbb{R}^{n\times n}$, $B(i) \in \mathbb{R}^{n\times n}$, $C_o(i) \in \mathbb{R}^{m\times n}$, $C_d(i) \in \mathbb{R}^{m\times n}$, $N(i) \in \mathbb{R}^{n\times r}$, $M(i) \in \mathbb{R}^{m\times r}$ and $L(i) \in \mathbb{R}^{p\times n}$ are known real constant matrices. $A_o(i)$, $A_d(i)$, $C_o(i)$, $C_d(i)$, $B(i)$, $N(i)$ and $M(i)$ are known real constant matrices of appropriate dimensions which describe the nominal system of $(\Sigma_{\Delta n})$. The matrices $\Delta A_o(t,\eta_t)$, $\Delta A_d(t,\eta_t)$, $\Delta C_o(t,\eta_t)$ and $\Delta C_d(t,\eta_t)$ are real, time-varying matrix functions representing the norm-bounded parameter uncertainties. For $\eta_t = i$, the admissible uncertainties are represented by

$$
\begin{bmatrix} \Delta A_o(t,i) & \Delta A_d(t,i) \\ \Delta C_o(t,i) & \Delta C_d(t,i) \end{bmatrix} = \begin{bmatrix} H_a(i) \\ H_c(i) \end{bmatrix} \Delta(t) [E_o(i) \quad E_d(i)],\quad \forall \Delta^{\mathrm{t}}(t)\Delta(t) \leqslant I,\ \forall t \qquad (2.8)
$$

where $H_a(i) \in \mathbb{R}^{n\times\alpha}$, $H_c(i) \in \mathbb{R}^{p\times\alpha}$, $E_o(i) \in \mathbb{R}^{\beta\times n}$ and $E_d(i) \in \mathbb{R}^{\beta\times n}$, are known real constant matrices and $\Delta(t) \in \mathbb{R}^{\alpha\times\beta}$ is an unknown matrix with Lebesgue measurable elements. The initial condition is specified as $\beta_o \triangleq \langle x(0), x(s)\rangle = \langle x_o, \phi(s)\rangle$, where $\phi(\cdot) \in \mathscr{L}_2[-\tau,0]$.

In the absence of uncertainties ($\Delta(\cdot) \equiv 0$) and for each possible value $\eta_{\mathrm{t}} = i$, $i \in \mathscr{S}$, we obtain the nominal neutral system

$$
(\Sigma_n)\quad \mathscr{M}(\dot{x}_t) \triangleq \dot{x}(t) - B(i)\dot{x}(t-\tau) = A_o(i)x(t) + A_d(i)x(t-\tau) + N(i)w(t), \qquad (2.9)
$$

$$
x(\eta) = \phi(\eta) \in \mathbb{C}([-\tau,0],\mathbb{R}^n),\quad \forall \eta \in [-\tau,0], \qquad (2.10)
$$

$$y(t) = C_o(i)x(t) + C_d(i)x(t-\tau) + M(i)w(t), \tag{2.11}$$
$$z(t) = L(i)x(t). \tag{2.12}$$

The following assumptions on systems $(\Sigma_{\Delta n})$ and (Σ_n) are recalled:

Assumption 2.1. $\lambda(A_o(i)) \in \mathbb{C}^-$, $i \in \mathscr{S}$.

Assumption 2.2. $|\lambda(B(i))| < 1$, $\det[B(i)] \neq 0$, $i \in \mathscr{S}$.

Remark 2.1. Note that system (2.3)–(2.6) is a hybrid system in which one state $x(t)$ takes values continuously, and another "state" $\eta(t)$ takes values discretely. Being continuous in time and represents a wide class of physical systems thus Assumption 2.1 is quite standard. On the other hand, Assumption 2.2 provides a sufficient condition on the eigenspectrum in the discrete space and its major role will be clarified in the sequel. An alternative interpretation of Assumption 2.2 is that the difference operator $\mathscr{M}(x_t)$ is delay-independently stable. The kind of systems (2.3)–(2.6) can be used to represent many important physical systems subject to random failures and structure changes, such as electric power systems [34], control systems of a solar thermal central receiver, communications systems, aircraft flight control, and manufacturing systems [3,25,14,33,6,8].

Our primary objective in this paper is to design robust state and robust $\mathscr{H}_\infty$ observers for the neutral system $(\Sigma_{\Delta n})$ with some desirable stability behavior and then extend these designs to the neutral system (Σ_n). Towards our goal, we let $\mathsf{X}(t, \beta_o, \eta_o)$ denote the trajectory of the state $x(t)$ from the initial state (β_o, η_o) and recall the following definition:

Definition 2.1. System $\Sigma_{\Delta n}$ is said to be robustly stochastically stable independent of delay (RSSID) if for all finite initial vector function $\phi(\cdot) \in \mathscr{L}_2[-\tau, 0]$ defined on the interval $[-\tau, 0]$ and initial mode $\eta_o \in \mathscr{S}$

$$\left\{ \int_0^\infty \mathbb{E}\left\{ \|\mathsf{X}(t, \beta_0, \eta_o)\|^2 \right\} \mathrm{d}t \right\} < +\infty \tag{2.13}$$

for all admissible uncertainties satisfying (2.8).

3. Robust observers

In the sequel, to derive the state estimate $\hat{x} \in \mathbb{R}^n$ we will utilize the following linear Markovian state-delayed observer for each possible value $\eta_t = i$, $i \in \mathscr{S}$

$$(\Sigma_e)\quad \mathscr{M}(\dot{\hat{x}}_t) \triangleq \dot{\hat{x}}(t) - B\dot{\hat{x}}(t-\tau) = A_f(i)\hat{x}(t) + A_d(i)\hat{x}(t-\tau) + K_f(i)y(t),\ \ \hat{x}(0)=0,$$
$$\hat{z}(t) = L(i)\hat{x}(t), \tag{3.1}$$

where $A_f(i) \in \mathbb{R}^{n\times n}$, $K_f(i) \in \mathbb{R}^{n\times m}$, $i \in \mathscr{S}$ are the observer matrix gains to be designed such that $\hat{x}$ reproduce x asymptotically for all admissible uncertainties satisfying (2.8).

3.1. The augmented system

Let the state error be

$$\tilde{x} \triangleq x(t) - \hat{x}(t). \tag{3.2}$$

From (2.3)–(2.5), (3.1) and (3.2), the state error dynamics can be represented by

$$\begin{aligned}(\Sigma_{\Delta e})\quad \mathscr{M}(\dot{\tilde{x}}_t) &\triangleq \dot{\tilde{x}}(t) - B(i)\dot{\tilde{x}}(t-\tau) = A_f(i)\tilde{x}(t) + A_d(i)\tilde{x}(t-\tau)\\ &+ [A_o(i) - K_f(i)C_o(i) - A_f(i)]x(t) + [\Delta A_o(t,i) - K_f(i)\Delta C_o(t,i)]x(t)\\ &- K_f(i)[C_d(i) + \Delta C_d(t,i)]x(t-\tau) + [N(i) - K_f(i)M(i)]w(t).\end{aligned} \tag{3.3}$$

A state-space augmented model of the observation error, $\tilde{z}(t) = z(t) - \hat{z}(t)$, can then be constructed in terms of the augmented state vector and the extended matrix $\mathbb{B}(i)$ for each possible value $\eta_t = i$, $i \in \mathscr{S}$

$$\zeta(t) \triangleq \begin{bmatrix}\tilde{x}(t)\\ x(t)\end{bmatrix}, \qquad \mathbb{B}(i) = \begin{bmatrix}B(i) & 0\\ 0 & B(i)\end{bmatrix} \tag{3.4}$$

by using (2.3)–(2.6), (3.3) and (3.4) as follows:

$$\begin{aligned}(\Sigma_{\Delta A})\quad \mathscr{M}(\dot{\zeta}_t) &\triangleq \dot{\zeta}(t) - \mathbb{B}(i)\dot{\zeta}(t-\tau)\\ &= \mathscr{A}_{\Delta o}(t,i)\zeta(t) + \mathscr{D}_{\Delta o}(t,i)\zeta(t-\tau) + \mathsf{B}(i)w(t),\end{aligned} \tag{3.5}$$
$$\tilde{z}(t) = L_f(i)\zeta(t), \tag{3.6}$$

where

$$\mathscr{A}_{\Delta o}(t,i) = [\mathsf{A}(i) + \overline{H}_A(i)\Delta(t,i)\overline{E}_A(i)], \quad \mathscr{D}_{\Delta o}(t,i) = [\mathsf{D}(i) + \overline{H}_D(i)\Delta(t,i)\overline{E}_D(i)],$$
$$L_f(i) = [L(i)\ \ 0], \quad \mathsf{A}(i) = \begin{bmatrix}A_f(i) & A_o(i) - K_f(i)C_o(i) - A_f(i)\\ 0 & A_o(i)\end{bmatrix},$$
$$\zeta(t-\tau) = \begin{bmatrix}\tilde{x}(t-\tau)\\ x(t-\tau)\end{bmatrix}, \quad \overline{E}_A(i) = [0\ \ E_a(i)], \quad \overline{E}_D(i) = [0\ \ E_d(i)], \tag{3.7}$$
$$\mathsf{D}(i) = \begin{bmatrix}A_d(i) & -K_f(i)C_d(i)\\ 0 & A_d(i)\end{bmatrix}, \quad \mathsf{B}(i) = \begin{bmatrix}N(i) - K_f(i)M(i)\\ N(i)\end{bmatrix}, \tag{3.8}$$
$$\overline{H}_A = \begin{bmatrix}H_a(i) - K_f(i)H_c(i)\\ H_a(i)\end{bmatrix}, \quad \overline{H}_D(i) = \begin{bmatrix}-K_f(i)H_c(i)\\ H_c(i)\end{bmatrix}. \tag{3.9}$$

Had we followed another route and combined systems (Σ_n) and (Σ_e), we would have obtained the nominal augmented system

$$(\Sigma_A) \quad \mathscr{M}(\dot{\zeta}_t) \triangleq \dot{\zeta}(t) - \mathbb{B}(i)\dot{\zeta}(t-\tau) = \mathsf{A}(i)\zeta(t) + \mathsf{D}(i)\zeta(t-\tau) + \mathsf{B}(i)w(t), \tag{3.10}$$

$$\tilde{z}(t) = L_f(i)\zeta(t). \tag{3.11}$$

Remark 3.1. It should be stressed that system $(\Sigma_{\Delta A})$ describes a linear uncertain jumping system of neutral-type the nominal version of which is represented by systems (Σ_A). The matrices of both systems depend on the gains $A_f(i)$, $K_f(i)$, $i \in \mathscr{S}$.

3.2. Stability analysis

The following theorems establish that the stability behavior of system $(\Sigma_{\Delta A})$ or (Σ_A) is related to the existence of a positive definite solution of linear matrix inequalities thereby providing a clear key to designing the state observers.

Theorem 3.1. *Given gain matrices $A_f(i)$, $K_f(i)$, $i \in \mathscr{S}$ and subject to Assumptions* 2.1 *and* 2.2, *system $(\Sigma_{\Delta A})$ with $w \equiv 0$ is (RSSID) if for given matrices $0 < \mathbb{L}(i) = \mathbb{L}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$, and letting*

$$\overline{\mathbb{L}}(i) = \mathbb{L}(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \quad \widehat{\mathbb{L}}(i) = \mathbb{L}(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m),$$
$$i \in \mathscr{S}$$

for some scalars $\xi(i) > 0$, $i \in \mathscr{S}$, there exist matrices $0 < \mathbb{P}(i) = \mathbb{P}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$ and scalars $\varepsilon(i) > 0$, $\varrho(i) > 0$, $i \in \mathscr{S}$ satisfying the following linear matrix inequalities (LMIs):

$$\begin{bmatrix} \Upsilon_t & \Lambda_a(i) & \Lambda_n(i) \\ \Lambda_a^{\mathrm{t}}(i) & -\Phi_a(i) & 0 \\ \Lambda_n^{\mathrm{t}}(i) & 0 & -\Theta_a(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.12}$$

$$\begin{bmatrix} -\overline{\mathbb{L}}(i) & \mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i) & \overline{E}_D^{\mathrm{t}}(i) & \varepsilon(i)\mathbb{B}^{\mathrm{t}}(i)\overline{E}_A^{\mathrm{t}}(i) \\ \widehat{\mathbb{L}}(i)\mathbb{B}(i) & \widehat{\mathbb{L}}(i) & 0 & 0 \\ \overline{E}_D(i) & 0 & -I & 0 \\ \varepsilon(i)\overline{E}_A(i)\mathbb{B}(i) & 0 & 0 & -\varepsilon(i)I \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.13}$$

$$\Upsilon_t = \mathbb{P}(i)\mathsf{A}(i) + \mathsf{A}^{\mathrm{t}}(i)\mathbb{P}(i) + \widehat{\mathbb{L}}(i) + \sum_{m=1}^{s} \alpha_{im}\mathbb{P}(m) + \varrho(i)\overline{E}_A^{\mathrm{t}}(i)\overline{E}_A(i),$$
$$\Lambda_a = \left[\, \mathbb{P}(i)\overline{H}_A(i) \quad \mathbb{P}(i)\overline{H}_A(i) \quad \mathbb{P}(i)\overline{H}_D(i) \,\right], \qquad \Phi_a = \mathrm{diag}[\varrho(i)I \quad \varepsilon(i)I \quad \varepsilon(i)I],$$
$$\Theta_a = \overline{\mathbb{L}}(i) - \mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i) - \varepsilon(i)\left(\mathbb{B}^{\mathrm{t}}(i)\overline{E}_A^{\mathrm{t}}(i)\overline{E}_A(i)\mathbb{B}(i) + \overline{E}_D^{\mathrm{t}}(i)\overline{E}_D(i)\right),$$
$$\Lambda_n(i) = \mathbb{P}(i)[\mathsf{A}(i)\mathbb{B}(i) + \mathsf{D}(i)] + \widehat{\mathbb{L}}(i)\mathbb{B}(i) \tag{3.14}$$

for all admissible uncertainties satisfying (2.8).

Proof. For $\eta_t = i$, $i \in \mathscr{S}$ and given $0 < \mathbb{L}(i) = \mathbb{L}^{\mathrm{t}}(i)$, let the Lyapunov functional $V(\cdot) : \mathbb{R}^n \times \mathbb{R}_+ \times \mathscr{S} \to \mathbb{R}_+$ be selected as

$$V(t,\zeta,\eta_t = i) \triangleq V(t,\zeta,i)$$
$$= \mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t) + \int_{-\tau}^{0} \zeta^{\mathrm{t}}(t+\theta)\mathbb{L}(i)\zeta(t+\theta)\,\mathrm{d}\theta. \tag{3.15}$$

The weak infinitesimal operator $\mathfrak{I}_a^{\zeta}[\cdot]$ of the process $\{\zeta(t), \eta_t, t \geqslant 0\}$ for system (3.5) at the point $\{t, \zeta, \eta_t\}$ is given by [15]

$$\mathfrak{I}_a^{\zeta}[V] = \frac{\partial V}{\partial t} + \mathscr{M}^{\mathrm{t}}(\dot{\zeta}_t)(t)\frac{\partial V}{\partial \zeta}\bigg|_{\eta_t=i} + \sum_{m=1}^{s} \alpha_{im} V(t,\zeta,i,m). \tag{3.16}$$

Using (3.5) into Eqs. (3.15) and (3.16) and manipulating the terms we get

$$\begin{aligned}
\mathfrak{I}_a^{\zeta}[V] = {} & \mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)[\mathscr{A}_{\Delta o}(t,i)\zeta(t) + \mathscr{D}_{\Delta o}(t,i)\zeta(t-\tau)] \\
& + [\mathscr{A}_{\Delta o}(t,i)\zeta(t) + \mathscr{D}_{\Delta o}(t,i)\zeta(t-\tau)]^{\mathrm{t}}\mathbb{P}\mathscr{M}(\zeta_t) + \mathscr{M}^{\mathrm{t}}(\zeta_t) \\
& \times \sum_{m=1}^{s} \alpha_{im}\mathbb{P}(m)\mathscr{M}(\zeta_t) + \zeta^{\mathrm{t}}(t)\mathbb{L}(i)\zeta(t) - \zeta^{\mathrm{t}}(t-\tau)\mathbb{L}(i)\zeta(t-\tau) \\
& + \sum_{m=1}^{s} \alpha_{im} \int_{-\tau}^{0} \zeta^{\mathrm{t}}(t+\theta)\mathbb{L}(m)\zeta(t+\theta)\,\mathrm{d}\theta \\
\leqslant {} & \mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)[\mathscr{A}_{\Delta o}(t,i)\zeta(t) + \mathscr{D}_{\Delta o}(t,i)\zeta(t-\tau)] \\
& + [\mathscr{A}_{\Delta o}(t,i)\zeta(t) + \mathscr{D}_{\Delta o}(t,i)\zeta(t-\tau)]^{\mathrm{t}}\mathbb{P}(i)\mathscr{M}(\zeta_t) + \mathscr{M}^{\mathrm{t}}(\zeta_t) \\
& \times \sum_{m=1}^{s} \alpha_{im}\mathbb{P}(m)\mathscr{M}(\zeta_t) + \zeta^{\mathrm{t}}(t)\widehat{\mathbb{L}}(i)\zeta(t) - \zeta^{\mathrm{t}}(t-\tau)\overline{\mathbb{L}}(i)\zeta(t-\tau),
\end{aligned} \tag{3.17}$$

where $\overline{\mathbb{L}}(i) > 0$, $i \in \mathscr{S}$ by selection of $\xi(i)$, $i \in \mathscr{S}$. Applying the argument of "completing the squares" and over-bounding the result, it yields

$$\begin{aligned}
\mathfrak{I}_a^{\zeta}[V] \leqslant\ & \mathscr{M}^{\mathrm{t}}(\zeta_t)\left[\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)+\mathscr{A}_{\Delta o}^{\mathrm{t}}(t,i)\mathbb{P}+\widehat{\mathbb{L}}(i)+\sum_{m=1}^{s}\alpha_{im}\mathbb{P}(m)\right]\mathscr{M}(\zeta_t)\\
&+\mathscr{M}^{\mathrm{t}}(\zeta_t)\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)\zeta(t-\tau)\\
&+\zeta^{\mathrm{t}}(t-\tau)\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)^{\mathrm{t}}\mathscr{M}(\zeta_t)\\
&-\zeta^{\mathrm{t}}(t-\tau)\Big[\overline{\mathbb{L}}(i)-\mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i)\Big]\zeta(t-\tau)\\
&\leqslant \mathscr{M}^{\mathrm{t}}(\zeta_t)\Bigg[\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)+\mathscr{A}_{\Delta o}^{\mathrm{t}}(t,i)\mathbb{P}(i)+\widehat{\mathbb{L}}(i)+\sum_{m=1}^{s}\alpha_{im}\mathbb{P}(m)\\
&+\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)\Big[\overline{\mathbb{L}}-\mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i)\Big]^{-1}\\
&\times\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)^{\mathrm{t}}\Bigg]\mathscr{M}(\zeta_t)\\
&-\Big[\zeta^{\mathrm{t}}(t-\tau)-\mathscr{M}^{\mathrm{t}}(\zeta_t)\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)\Big]\\
&\times\Big[\overline{\mathbb{L}}(i)-\mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i)\Big]^{-1}\Big[\zeta(t-\tau)-(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)\\
&+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i))^{\mathrm{t}}\mathscr{M}(\zeta_t)\Big]\leqslant \mathscr{M}^{\mathrm{t}}(\zeta_t)\Bigg[\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)+\mathscr{A}_{\Delta o}^{\mathrm{t}}(t,i)\mathbb{P}(i)+\widehat{\mathbb{L}}(i)\\
&+\sum_{m=1}^{s}\alpha_{im}\mathbb{P}(m)+\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)\\
&\times\Big[\overline{\mathbb{L}}-\mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i)\Big]^{-1}\Big(\mathbb{P}(i)\mathscr{A}_{\Delta o}(t,i)\mathbb{B}(i)+\widehat{\mathbb{L}}(i)\mathbb{B}(i)+\mathbb{P}(i)\mathscr{D}_{\Delta o}(t,i)\Big)^{\mathrm{t}}\Bigg]\mathscr{M}(\zeta_t).
\end{aligned}\tag{3.18}$$

Using Facts 2 and 3, it follows from (3.18) for some scalars $\varepsilon(i) > 0$, $\varrho(i) > 0$, $i \in \mathscr{S}$ that:

$$\begin{aligned}
\mathfrak{I}_a^{\zeta}[V] \leqslant\ & \mathscr{M}^{\mathrm{t}}(\zeta_t)\Bigg\{\mathbb{P}(i)\mathsf{A}(i)+\mathsf{A}^{\mathrm{t}}(i)\mathbb{P}(i)+\widehat{\mathbb{L}}(i)+\sum_{m=1}^{s}\alpha_{im}\mathbb{P}(m)\\
&+\varrho^{-1}(i)\mathbb{P}(i)\overline{H}_A(i)\overline{H}_A^{\mathrm{t}}(i)\mathbb{P}(i)+\varrho(i)\overline{E}_A^{\mathrm{t}}(i)\overline{E}_A(i)\\
&+\varepsilon^{-1}(i)\mathbb{P}(i)\Big[\overline{H}_A(i)\overline{H}_A^{\mathrm{t}}(i)+\overline{H}_D(i)\overline{H}_D^{\mathrm{t}}(i)\Big]\mathbb{P}(i)\\
&+\Big(\mathbb{P}(i)[\mathsf{A}(i)\mathbb{B}(i)+\mathsf{D}(i)]+\widehat{\mathbb{L}}(i)\mathbb{B}(i)\Big)\\
&\times\Big[\overline{\mathbb{L}}-\mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i)-\varepsilon(i)\Big(\overline{E}_A^{\mathrm{t}}(i)\overline{E}_A(i)+\overline{E}_D^{\mathrm{t}}(i)\overline{E}_D(i)\Big)\Big]^{-1}\\
&\times\Big(\mathbb{P}(i)[\mathsf{A}(i)\mathbb{B}(i)+\mathsf{D}(i)]+\widehat{\mathbb{L}}(i)\mathbb{B}(i)\Big)^{\mathrm{t}}\Bigg\}\mathscr{M}(\zeta_t)\triangleq\mathscr{M}^{\mathrm{t}}(\zeta_t)\Pi(i)\mathscr{M}(\zeta_t).
\end{aligned}\tag{3.19}$$

By the Schur complements, inequality (3.19) is equivalent to LMIs (3.12) and (3.13) from which we conclude that for all admissible uncertainties satisfying (2.8) $\mathfrak{I}_a^{\zeta}[V] < 0$ for all $\zeta \neq 0$ and $\mathfrak{I}_a^{\zeta}[V] \leqslant 0$ for all ζ.

Since $\|x(t+\beta)\| \leqslant \varphi\|x(t)\|, \forall \beta \in [-\tau, 0]$ and some $\varphi > 0$ [17], it follows from (3.15) that

$$V(t,\zeta,i) \leqslant \mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t) + \mu\|\zeta\|^2,$$
$$\mu = \varphi\tau\left(\max_i \lambda_{\mathrm{M}}[\mathbb{P}(i)] + \lambda_{\mathrm{M}}[\mathbb{L}(i)]\right).$$

Therefore, for all $\zeta \neq 0$, we have

$$\frac{\mathfrak{I}_a^{x}[V]}{V(t,\zeta,i)} \leqslant \frac{\mathscr{M}^{\mathrm{t}}(\zeta_t)\Pi(i)\mathscr{M}(\zeta_t)}{\mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t)} \leqslant -\sigma \triangleq -\min_{i\in\mathscr{S}}\left\{\frac{\lambda_{\mathrm{m}}[-\Pi(i)]}{\lambda_{\mathrm{M}}[\mathbb{P}(i)]}\right\}. \tag{3.20}$$

It is readily seen from (3.20) that $\sigma > 0$ and hence we get $\mathfrak{I}_a^{\zeta}[V] \leqslant -\sigma V(t,\zeta,i)$. Then, it follows from [15], by using the Gronwall–Bellman lemma [20] and letting $x(t=0,\phi,i) = x_o$, that

$$\mathbb{E}[V(t,\zeta,i)|\phi,\eta_o] \leqslant e^{-\sigma t}V(t,\zeta_o,\eta_o) \tag{3.21}$$

Therefore

$$\begin{aligned}\mathbb{E}[V(t,\zeta,i)|\phi,\eta_o] &= \mathbb{E}\left\{\mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t) + \int_{-\tau}^{0}\zeta^{\mathrm{t}}(t+\theta)\mathbb{L}(i)\zeta(t+\theta)\,\mathrm{d}\theta|\phi,\eta_o\right\}\\ &= \mathbb{E}\{\mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t)|\phi,\eta_o\}\\ &\quad + \mathbb{E}\left\{\int_{-\tau}^{0}\zeta^{\mathrm{t}}(t+\theta)\mathbb{L}(i)\zeta(t+\theta)\,\mathrm{d}\theta|\phi,\eta_o\right\}\\ &\leqslant e^{-\sigma t}V(t,\zeta_o,\eta_o).\end{aligned} \tag{3.22}$$

Since $\mathbb{E}\left\{\int_{-\tau}^{0}\zeta^{\mathrm{t}}(t+\theta)\mathbb{L}(i)\zeta(t+\theta)\,\mathrm{d}\theta|\phi,\eta_o\right\} \geqslant 0$; then some algebraic manipulation of (3.22) yields:

$$\begin{aligned}&\mathbb{E}\{\mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t)|\phi,\eta_o\} \leqslant e^{-\sigma t}V(t,\zeta_o,\eta_o)\\ &\quad\Rightarrow\ \mathbb{E}\left\{\int_0^{\mathscr{T}}\mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t)\,\mathrm{d}t|\phi,\eta_o\right\}\\ &\qquad\qquad \leqslant \left[\int_0^{\mathscr{T}} e^{-\sigma t}\,\mathrm{d}t\right]V(x_o,\eta_o) = \frac{1}{\sigma}[e^{-\zeta\mathscr{T}} - 1]V(t,\zeta_o,\eta_o)\\ &\quad\Rightarrow\ \lim_{\mathscr{T}\to\infty}\mathbb{E}\left\{\int_0^{\mathscr{T}}\mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}(i)\mathscr{M}(\zeta_t)\,\mathrm{d}t|\phi,\eta_o\right\}\\ &\qquad\qquad \leqslant \frac{1}{\sigma}\zeta_o^{\mathrm{t}}\mathbb{P}(\eta_o)\zeta_o + \frac{\tau}{\sigma}[\mathbb{L}(\eta_o)]\|\zeta(t+\theta)\|_*^2, \quad \forall\theta \in [-\tau, 0],\end{aligned} \tag{3.23}$$

where $\|x(t+\theta)\|_*^2 \triangleq \sup_{\theta\in[-\tau,0]}\|\zeta(t+\theta)\|_2^2$. Now, let

$$\overline{\mathbb{P}}(i) = \max_{i\in\mathscr{S}}\left\{\frac{\lambda_{\mathrm{M}}[\mathbb{P}(\eta_o)]\|\zeta_o\|^2 + \tau[\mathbb{L}(\eta_o)]\|x(t+\theta)\|_*^2}{\sigma[\mathbb{P}(\eta_o)]\|\zeta_o\|^2}\right\},$$

it follows from (3.23) for $i\in\mathscr{S}$ that

$$\lim_{\mathscr{T}\to\infty}\mathbb{E}\left\{\int_0^{\mathscr{T}}\zeta^{\mathrm{t}}(t)\zeta(t)\,\mathrm{d}t|\phi,\eta_o\right\} \leqslant \zeta_o^{\mathrm{t}}\lambda_{\mathrm{M}}(\overline{\mathbb{P}}(i))\zeta_o < +\infty,$$

which, in the light of Definition 2.1, shows that system $(\Sigma_{\Delta A})$ is (RSSID). □

Theorem 3.2. *Given gain matrices* $A_f(i)$, $K_f(i)$, $i\in\mathscr{S}$ *and subject to Assumptions* 2.1 *and* 2.2, *system* (Σ_A) *with* $w\equiv 0$ *is stochastically stable independent of delay* (*SSID*) *if for given matrices* $0<\mathbb{L}(i)=\mathbb{L}^{\mathrm{t}}(i)\in\mathbb{R}^{2n\times 2n}$, $i\in\mathscr{S}$, *and letting*

$$\overline{\mathbb{L}}(i) = \mathbb{L}(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \quad \widehat{\mathbb{L}}(i) = \mathbb{L}(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \qquad i\in\mathscr{S}$$

for some scalars $\xi(i)>0$, $i\in\mathscr{S}$, *there exist matrices* $0<\mathbb{P}(i)=\mathbb{P}^{\mathrm{t}}(i)\in\mathbb{R}^{2n\times 2n}$, $i\in\mathscr{S}$ *satisfying the following LMIs*:

$$\begin{bmatrix}\Upsilon_{to} & \Lambda_n(i)\\ \Lambda_n^{\mathrm{t}}(i) & -\Theta_{ao}(i)\end{bmatrix}<0, \quad \begin{bmatrix}-\overline{\mathbb{L}}(i) & \mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\\ \widehat{\mathbb{L}}(i)\mathbb{B}(i) & \widehat{\mathbb{L}}(i)\end{bmatrix}<0, \qquad i\in\mathscr{S} \tag{3.24}$$

$$\Upsilon_{to} = \mathbb{P}(i)\mathsf{A}(i) + \mathsf{A}^{\mathrm{t}}(i)\mathbb{P}(i) + \widehat{\mathbb{L}}(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}(m),$$

$$\Theta_{ao} = \overline{\mathbb{L}}(i) - \mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i). \tag{3.25}$$

Proof. Follows from Theorem 3.1 by setting $\overline{E}_A^{\mathrm{t}}\equiv 0$, $\overline{E}_D^{\mathrm{t}}\equiv 0$, $\overline{H}_A^{\mathrm{t}}\equiv 0$. □

Remark 3.2. It should be remarked that both Theorems 3.1 and 3.2 offer new analytical results for the class of neutral-type dynamical systems under consideration. The results are cast in LMI format for which the MATLAB-LMI software is readily available [10]. More importantly, in the case $B(i)\equiv 0 \Rightarrow \mathscr{M}(\cdot)=x_t$, systems $(\Sigma_{\Delta A})$ and (Σ_A) become of retarded-type for which Theorems 3.1 and 3.2 retrieve the results of [29,22].

Remark 3.3. The need for Assumption 2.2 is evident from (3.14) and (3.25) in which case the conditions $\Theta_a>0$, $\Theta_{ao}>0$ are required, respectively. In both cases, the result reveals a discrete Lyapunov inequality.

3.3. Observer design

In this section, we provide expressions for the gain matrices of the observer (3.1) when applied to the neural systems ($\Sigma_{\Delta A}$) and (Σ_A). To facilitate further development, we introduce the following matrix expressions for some scalars $\varepsilon(i) > 0$, $\varrho(i) > 0$, $i \in \mathscr{S}$:

$$\mathbb{P}(i) = \begin{bmatrix} \mathbb{P}_e(i) & 0 \\ 0 & \mathbb{P}_x(i) \end{bmatrix}, \quad \mathbb{L}(i) = \begin{bmatrix} \mathbb{L}_e(i) & 0 \\ 0 & \mathbb{L}_x(i) \end{bmatrix}, \tag{3.26}$$

$$\begin{aligned}
\Xi_a(i) &= \overline{\mathbb{L}}_e(i) - B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_e(i)B(i), \\
\Xi_b(i) &= \overline{\mathbb{L}}_x(i) - B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i)B(i) - \varepsilon(i)(E_a^{\mathrm{t}}(i)E_a(i) + E_d^{\mathrm{t}}(i)E_d(i)), \\
\Xi_c(i) &= B(i)\Xi_b^{-1}(i)[B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i) + [A_d^{\mathrm{t}}(i) + B^{\mathrm{t}}(i)A_o^{\mathrm{t}}(i)]\mathbb{P}_x(i)],
\end{aligned} \tag{3.27}$$

$$\begin{aligned}
\widehat{A}_o(i) &= A_o(i) + \varrho^{-1}(i)H_a(i)H_a^{\mathrm{t}}(i)\mathbb{P}_x(i)(I + \Xi_c(i))^{-1}, \\
\widehat{C}_o(i) &= C_o(i) + [C_d(i)B^{-1}(i)\Xi_c(i) + (\varepsilon^{-1}(i) + \varrho^{-1}(i))H_c(i)H_a^{\mathrm{t}}(i)\mathbb{P}_x(i) \\
&\quad + \varepsilon^{-1}(i)H_c(i)H_c^{\mathrm{t}}(i)\mathbb{P}_x(i)](I + \Xi_c(i))^{-1},
\end{aligned} \tag{3.28}$$

$$\begin{aligned}
\Xi_k(i) &= (2\varepsilon^{-1}(i) + \varrho^{-1}(i))H_c(i)H_c^{\mathrm{t}}(i) + \widehat{C}_o(i)B(i)\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\widehat{C}_o^{\mathrm{t}}(i) \\
&\quad + [[\widehat{C}_o(i) + C_o(i)]B(i) + C_d(i)]\Xi_b^{-1}(i)[[\widehat{C}_o(i) + C_o(i)]B(i) + C_d(i)]^{\mathrm{t}},
\end{aligned} \tag{3.29}$$

$$\begin{aligned}
\Xi_g(i) &= (\varepsilon^{-1}(i) + \varrho^{-1}(i))H_a(i)H_c^{\mathrm{t}}(i) + (\widehat{A}_o(i) + A_d(i) + \widehat{\mathbb{L}}_e(i)B(i))\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\widehat{C}_o(i) \\
&\quad + (\widehat{A}_o(i) - A_o(i) + \mathbb{L}_e(i)B(i))\Xi_b^{-1}(i)[[\widehat{C}_o(i) + C_o(i)]B(i) + C_d(i)]^{\mathrm{t}}.
\end{aligned} \tag{3.30}$$

The main results are summarized by the following theorems:

Theorem 3.3. *System $\Sigma_{\Delta A}$ is RSSID, if given matrices $0 < \mathbb{L}_e(i) = \mathbb{L}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{L}_x(i) = \mathbb{L}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$ and scalars $\xi(i) > 0$ such that*

$$\overline{\mathbb{L}}_x(i) = \mathbb{L}_x(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad \widehat{\mathbb{L}}_x(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad i \in \mathscr{S},$$

$$\overline{\mathbb{L}}_e(i) = \mathbb{L}_e(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad \widehat{\mathbb{L}}_e(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad i \in \mathscr{S},$$

there exist matrices $0 < \mathbb{P}_e(i) = \mathbb{P}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{P}_x(i) = \mathbb{P}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $i \in \mathscr{S}$ and scalars $\varepsilon(i) > 0$, $\varrho(i) > 0$ satisfying the LMIs

$$\begin{bmatrix} \Upsilon_e(i) & \Lambda_b(i) & \Lambda_d(i) & \Lambda_f(i) \\ \Lambda_b^{\mathrm{t}}(i) & -\Phi_b(i) & 0 & 0 \\ \Lambda_d^{\mathrm{t}}(i) & 0 & -\Xi_a(i) & 0 \\ \Lambda_f^{\mathrm{t}}(i) & 0 & 0 & -\Xi_b(i) \end{bmatrix} < 0, \quad \begin{bmatrix} \Upsilon_x(i) & \Lambda_c(i) & \Lambda_h(i) \\ \Lambda_c^{\mathrm{t}}(i) & -\Phi_c(i) & 0 \\ \Lambda_h^{\mathrm{t}}(i) & 0 & -\Xi_b(i) \end{bmatrix} < 0, \tag{3.31}$$

where

$$\begin{aligned}
\Upsilon_e(i) &= \mathbb{P}_e(i)\widehat{A}_o(i) + \widehat{A}_o^{\mathrm{t}}(i)\mathbb{P}_e(i) + \widehat{\mathbb{L}}_e(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_e(m) - \mathbb{P}_e\Xi_g^{\mathrm{t}}\Xi_k^{-1}\Xi_g\mathbb{P}_e,\\
\Lambda_b(i) &= [\,\mathbb{P}_e(i)H_a(i) \quad \mathbb{P}_e(i)H_a(i)\,],\\
\Phi_b(i) &= \mathrm{diag}[\,\varepsilon(i)I \quad \varrho(i)I\,],\\
\Lambda_d(i) &= \mathbb{P}_e(i)[\widehat{A}_o(i) + A_d(i)] + \widehat{\mathbb{L}}_e(i)B(i),\\
\Lambda_f(i) &= \mathbb{P}_e(i)[\widehat{A}_o(i) - A_o(i)] + \widehat{\mathbb{L}}_e(i)B(i),\\
\Upsilon_x(i) &= \mathbb{P}_x(i)A_o(i) + A_o^{\mathrm{t}}(i)\mathbb{P}_x(i) + \widehat{\mathbb{L}}_x(i) + \varrho E_a^{\mathrm{t}}(i)E_a(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_x(m),
\end{aligned} \tag{3.32}$$

$$\begin{aligned}
\Lambda_h(i) &= \mathbb{P}_x(i)A_o(i)B(i) + \mathbb{P}_x(i)A_d(i) + \widehat{\mathbb{L}}_x(i)B(i),\\
\Lambda_c(i) &= [\,\mathbb{P}_e(i)H_a(i) \quad \mathbb{P}_e(i)H_a(i) \quad \mathbb{P}_x(i)H_c(i)\,],\\
\Phi_c(i) &= \mathrm{diag}[\,\varepsilon(i)I \quad \varrho(i)I \quad \varepsilon(i)I\,]
\end{aligned} \tag{3.33}$$

for all admissible uncertainties satisfying (2.8). *Moreover*, *the estimator gains are given by*

$$K_f(i) = \Xi_g(i)\Xi_k^{-1}(i), \qquad A_f(i) = \widehat{A}_o(i) - K_f(i)\widehat{C}_o(i). \tag{3.34}$$

Proof. Extending on Theorem 3.2 by using (3.7)–(3.9) and (3.26)–(3.30) into (3.19) and expanding terms we express the result into the block form

$$\begin{bmatrix} \Sigma_{11}(i) & \Sigma_{12}(i) \\ \Sigma_{12}^{\mathrm{t}}(i) & \Sigma_{22}(i) \end{bmatrix},$$

where

$$\begin{aligned}
\Sigma_{11}(i) = {} & \mathbb{P}_e(i)A_f(i) + A_f^{\mathrm{t}}(i)\mathbb{P}_e(i) + \widehat{\mathbb{L}}_e(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_e(m)\\
& + (\varepsilon^{-1}(i) + \varrho^{-1}(i))\mathbb{P}_e(i)[H_a(i) - K_f(i)H_c(i)][H_a(i) - K_f(i)H_c(i)]^{\mathrm{t}}\mathbb{P}_e(i)\\
& + \varepsilon^{-1}(i)\mathbb{P}_e(i)K_f(i)H_c(i)H_c^{\mathrm{t}}(i)K_f^{\mathrm{t}}(i)\mathbb{P}_e(i)\\
& + [\mathbb{P}_e(i)[A_f(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_e(i)B(i)]\Xi_a^{-1}(i)\\
& \cdot [\mathbb{P}_e(i)[A_f(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_e(i)B(i)]^{\mathrm{t}}\\
& + [\mathbb{P}_e(i)[A_o(i) - A_f(i) - K_f(i)C_o(i)]B(i) - \mathbb{P}_e(i)K_f(i)C_d(i)]\Xi_b^{-1}(i)\\
& \cdot [\mathbb{P}_e(i)[A_o(i) - A_f(i) - K_f(i)C_o(i)]B(i) - \mathbb{P}_e(i)K_f(i)C_d(i)]^{\mathrm{t}},
\end{aligned} \tag{3.35}$$

$$
\begin{aligned}
\Sigma_{22}(i) = {} & \mathbb{P}_x(i)A_o(i) + A_o^{\mathrm{t}}(i)\mathbb{P}_x(i) + \widehat{\mathbb{L}}_x(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_x(m) + \varrho(i)E_a^{\mathrm{t}}(i)E_a(i) \\
& + (\varepsilon^{-1}(i) + \varrho^{-1}(i))\mathbb{P}_x(i)H_a(i)H_a^{\mathrm{t}}(i)\mathbb{P}_x(i) + \varepsilon^{-1}(i)\mathbb{P}_x(i)H_c(i)H_c^{\mathrm{t}}(i)\mathbb{P}_x(i) \\
& + [\mathbb{P}_x(i)[A_o(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_x(i)B(i)]\Xi_b^{-1}(i) \\
& \cdot [\mathbb{P}_x(i)[A_o(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_x(i)B(i)]^{\mathrm{t}}, \qquad (3.36) \\
\Sigma_{12}(i) = {} & [\mathbb{P}_e(i)[A_o(i) - A_f(i) - K_f(i)C_o(i)]B(i) - \mathbb{P}_e(i)K_f(i)C_d(i)]\Xi_b^{-1}(i) \\
& \cdot [\mathbb{P}_x(i)[A_o(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_x(i)B(i)]^{\mathrm{t}} \\
& - \varepsilon^{-1}\mathbb{P}_e(i)K_f(i)H_c(i)H_c^{\mathrm{t}}(i)K_f^{\mathrm{t}}(i)\mathbb{P}_x(i) + \mathbb{P}_e(i)[A_o(i) - A_f(i) - K_f(i)C_o(i)] \\
& + (\varepsilon^{-1}(i) + \varrho^{-1}(i))\mathbb{P}_e(i)[H_a(i) - K_f(i)H_c(i)]H_a^{\mathrm{t}}(i)\mathbb{P}_x(i). \qquad (3.37)
\end{aligned}
$$

Applying Fact 1 to the matrix block Σ_{22}, we can readily obtain one of the LMIs (3.31). The substitution of (3.28)–(3.30) into (3.37) renders $\Sigma_{12} \equiv 0$. Using (3.28)–(3.30) and (3.34) into (3.35) with some matrix manipulations and applying the Schur complements we get the other LMI (3.31). □

Theorem 3.4. *System Σ_A is* (SSID), *if given matrices* $0 < \mathbb{L}_e(i) = \mathbb{L}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{L}_x(i) = \mathbb{L}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$ *and scalars* $\xi(i) > 0$ *such that*

$$
\overline{\mathbb{L}}_x(i) = \mathbb{L}_x(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad \widehat{\mathbb{L}}_x(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad i \in \mathscr{S},
$$

$$
\overline{\mathbb{L}}_e(i) = \mathbb{L}_e(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad \widehat{\mathbb{L}}_e(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad i \in \mathscr{S}
$$

there exist matrices $0 < \mathbb{P}_e(i) = \mathbb{P}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{P}_x(i) = \mathbb{P}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $i \in \mathscr{S}$ *satisfying the LMIs*

$$
\begin{bmatrix} \Upsilon_{eo}(i) & \Lambda_{do}(i) & \widehat{\mathbb{L}}_e(i)B(i) \\ \Lambda_{do}^{\mathrm{t}}(i) & -\Xi_a(i) & 0 \\ B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_e^{\mathrm{t}}(i) & 0 & -\Xi_{bo}(i) \end{bmatrix} < 0, \quad \begin{bmatrix} \Upsilon_{xo}(i) & \Lambda_h(i) \\ \Lambda_h^{\mathrm{t}}(i) & -\Xi_{bo}(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}, \qquad (3.38)
$$

where

$$
\begin{aligned}
\Upsilon_{eo}(i) &= \mathbb{P}_e(i)A_o(i) + A_o^{\mathrm{t}}(i)\mathbb{P}_e(i) + \widehat{\mathbb{L}}_e(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_e(m) - \mathbb{P}_e\Xi_{go}^{\mathrm{t}}\Xi_{ko}^{-1}\Xi_{go}\mathbb{P}_e \\
\Lambda_{do}(i) &= \mathbb{P}_e(i)[A_o(i) + A_d(i)] + \widehat{\mathbb{L}}_e(i)B(i) \\
\Upsilon_{xo}(i) &= \mathbb{P}_x(i)A_o(i) + A_o^{\mathrm{t}}(i)\mathbb{P}_x(i) + \widehat{\mathbb{L}}_x(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_x(m) \\
\Lambda_h(i) &= \mathbb{P}_x(i)A_o(i)B(i) + \mathbb{P}_x(i)A_d(i) + \widehat{\mathbb{L}}_x(i)B(i). \qquad (3.39)
\end{aligned}
$$

Moreover, the estimator gains are given by

$$K_f(i) = \Xi_{go}(i)\Xi_{ko}^{-1}(i), \qquad A_f(i) = A_o(i) - K_f(i)\overline{C}_o(i) \tag{3.40}$$

Proof. Define

$$\begin{aligned}
\Xi_{bo}(i) &= \overline{\mathbb{L}}_x(i) - B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i)B(i),\\
\Xi_{co}(i) &= B(i)\Xi_{bo}^{-1}(i)\Big[B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i) + [A_d^{\mathrm{t}}(i) + B^{\mathrm{t}}(i)A_o^{\mathrm{t}}(i)]\mathbb{P}_x(i)\Big],\\
\overline{C}_o(i) &= C_o(i) + C_d(i)B^{-1}(i)\Xi_{co}(i)(I + \Xi_{co}(i))^{-1},\\
\Xi_{ko}(i) &= \overline{C}_o(i)B(i)\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\overline{C}_o^{\mathrm{t}}(i)\\
&\quad + \big[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\big]\Xi_{bo}^{-1}(i)\big[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\big]^{\mathrm{t}},\\
\Xi_{go}(i) &= (A_o(i) + A_d(i) + \widehat{\mathbb{L}}_e(i)B(i))\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\overline{C}_o(i)\\
&\quad + \widehat{\mathbb{L}}_e(i)B(i)\Xi_{co}^{-1}(i)\big[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\big]^{\mathrm{t}}.
\end{aligned} \tag{3.41}$$

By setting $\varrho \equiv 0$, $\varepsilon \equiv 0$, $H_a \equiv 0$, $H_a \equiv 0$, $E_a \equiv 0$, $H_c \equiv 0$, $E_d \equiv 0$ while using (3.41) and following similar technique to Theorem 3.3, the desired result is achieved. □

3.4. $\mathscr{H}_\infty$ performance

In order to improve the foregoing robust observer results further, one would direct the design effort on robust observation in an $\mathscr{H}_\infty$ setting. For this purpose we recall the following definition:

Definition 3.1. System $(\Sigma_{\Delta n})$ is said to be RSSID with a disturbance attenuation γ if for all finite initial vector function $\phi(\cdot) \in \mathscr{L}_2[-\tau, 0]$ defined on the interval $[-\tau, 0]$ and initial mode $\eta_o \in \mathscr{S}$

$$\|\tilde{z}(t)\|_{E_2} \triangleq \mathbb{E}\left[\int_0^\infty \tilde{z}^{\mathrm{t}}(t)\tilde{z}(t)\,\mathrm{d}t\right]^{1/2} < \gamma\|w(t)\|_2$$

for all $0 \neq w(t) \in \mathscr{L}_2[0, \infty)$ and for all admissible uncertainties satisfying (2.8).

Our objective in this section is to design robust observers for the neutral system $(\Sigma_{\Delta n})$ with some desirable stability behavior and guaranteed $\mathscr{H}_\infty$ performance and then extend this design to the neutral system (Σ_n). Based thereon, the following theorems are established:

Theorem 3.5. *Given gain matrices* $A_f(i)$, $K_f(i)$, $i \in \mathscr{S}$ *and subject to Assumptions* 2.1 *and* 2.2, *system* $(\Sigma_{\Delta A})$ *is RSSID with a disturbance attenuation* γ *if given matrices* $0 < \mathbb{L}(i) = \mathbb{L}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$ *and scalars* $\xi(i) > 0$ *such that*

$$\overline{\mathbb{L}}(i) = \mathbb{L}(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad \widehat{\mathbb{L}}(i) = \mathbb{L}(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \qquad i \in \mathscr{S}$$

there exist matrices $0 < \mathbb{P}(i) = \mathbb{P}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$ *and scalars* $\gamma > 0$, $\varepsilon(i) > 0$, $\varrho(i) > 0$ *satisfying the LMIs*:

$$\begin{bmatrix} \Upsilon_w(i) & \Lambda_a(i) & \Gamma_a(i) & \mathbb{P}(i)\mathsf{B}(i) \\ \Lambda_a^{\mathrm{t}}(i) & -\Phi_a(i) & 0 & 0 \\ \Gamma_a^{\mathrm{t}}(i) & 0 & -\Theta_b(i) & 0 \\ \mathsf{B}^{\mathrm{t}}(i)\mathbb{P}(i) & 0 & 0 & -\gamma^2 I + L_f^{\mathrm{t}}(i)L_f(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.42}$$

$$\begin{bmatrix} -\overline{\mathbb{L}}(i) & \mathbb{B}^{\mathrm{t}}(i)L_f^{\mathrm{t}}(i) & \mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i) & \overline{E}_D^{\mathrm{t}}(i) & \varepsilon\mathbb{B}^{\mathrm{t}}(i)\overline{E}_A^{\mathrm{t}}(i) \\ L_f(i)\mathbb{B}(i) & -I & 0 & 0 & 0 \\ \widehat{\mathbb{L}}(i)\mathbb{B}(i) & 0 & -\widehat{\mathbb{L}}(i) & 0 & 0 \\ \overline{E}_D(i) & 0 & 0 & -I & 0 \\ \varepsilon\overline{E}_A\mathbb{B} & 0 & 0 & 0 & -\varepsilon I \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.43}$$

where

$$\Upsilon_w(i) = \Upsilon_t(i) + L_f^{\mathrm{t}}(i)L_f(i), \Gamma_a(i) = \Lambda_n(i) + L_f^{\mathrm{t}}(i)L_f(i),$$
$$\Theta_b(i) = \Theta_a(i) + L_f^{\mathrm{t}}(i)L_f(i) \tag{3.44}$$

for all admissible uncertainties satisfying (2.8).

Proof. The asymptotic stability follows from Theorem 3.1. Let stochastic Lyapunov functional $V(t,\zeta,i)$ be given by (3.15). The weak infinitesimal operator $\mathfrak{I}_w^{\zeta}[\cdot]$ of the process $\{x(t),\eta_t,t \geqslant 0\}$ for system (3.8) at the point $\{t,x,\eta_t\}$ is given by

$$\mathfrak{I}_w^{\zeta}[V] = \mathfrak{I}_a^{\zeta}[V] + \mathscr{M}^{\mathrm{t}}(\zeta_t)\mathbb{P}\mathsf{B}w(t) + w^{\mathrm{t}}(t)\mathsf{B}^{\mathrm{t}}\mathbb{P}\mathscr{M}(\zeta_t). \tag{3.45}$$

Introduce the performance measure

$$\mathscr{J}(\zeta) := \mathbb{E}\left\{\int_0^{\infty}\left[\tilde{z}^{\mathrm{t}}(t)\tilde{z}(t) - \gamma^2 w^{\mathrm{t}}(t)w(t)\right]\mathrm{d}t\right\}.$$

By Dynkin's formula [15], one has

$$\mathbb{E}\left\{\int_0^{\infty}\mathfrak{I}_w^{\zeta}[V]\,\mathrm{d}t\right\} = \mathbb{E}\{V(t,x,\eta_t)|_{t=\infty}\} - V(t,x,\eta_t)|_{t=0} \geqslant 0.$$

With some manipulations using (3.8) and (3.19), we obtain

$$\begin{aligned}\mathscr{J}(\zeta) &= \mathbb{E}\left\{\int_0^{\infty}\left[\tilde{z}^{t}(t)\tilde{z}(t) - \gamma^2 w^{t}(t)w(t) + \mathfrak{I}_w^{\zeta}[V] - \mathfrak{I}_w^{\zeta}[V]\right]\mathrm{d}t\right\}\\ &\leqslant \mathbb{E}\left\{\int_0^{\infty}\left[\mathscr{M}^{t}(\zeta_t)\mathbb{P}\mathsf{B}w(t) + w^{t}(t)\mathsf{B}^{t}\mathbb{P}\mathscr{M}(\zeta_t) - w^{t}(t)[\gamma^2 I\right.\right.\\ &\quad \left.\left. -L_f^{t}(i)L_f(i)]w(t) + \mathscr{M}^{t}(\zeta_t)\Pi(i)\mathscr{M}(\zeta_t)\right]\mathrm{d}t\right\}\\ &\leqslant \mathbb{E}\left\{\int_0^{\mathscr{T}}\mathscr{M}^{t}(\zeta_t)\left(\Pi(i) + \mathbb{P}\mathsf{B}[\gamma^2 I - L_f^{t}(i)L_f(i)]^{-1}\mathsf{B}^{t}\mathbb{P}\right)\mathscr{M}(\zeta_t)\,\mathrm{d}t\right\}.\end{aligned} \tag{3.46}$$

By using (3.43) together with Fact 1 and the results of Theorem 3.1, it follows from inequality (3.46) that $\mathscr{J}(\zeta) < 0$ for all admissible uncertainties satisfying (2.8). Hence, by Definition 3.1, the proof is completed. □

Theorem 3.6. *Given gain matrices $A_f(i)$, $K_f(i)$, $i \in \mathscr{S}$ and subject to Assumptions* 2.1 *and* 2.2, *system* (Σ_A) *is SSID with a disturbance attenuation* γ *if given matrices* $0 < \mathbb{L}(i) = \mathbb{L}^{t}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$ *and scalars* $\xi(i) > 0$ *such that*

$$\overline{\mathbb{L}}(i) = \mathbb{L}(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \quad \widehat{\mathbb{L}}(i) = \mathbb{L}(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \qquad i \in \mathscr{S}$$

there exist matrices $0 < \mathbb{P}(i) = \mathbb{P}^{t}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$ *satisfying the following LMIs*:

$$\begin{bmatrix} \Upsilon_w(i) & \Gamma_a(i) & \mathbb{P}(i)\mathsf{B}(i) \\ \Gamma_a^{t}(i) & -\Theta_b(i) & 0 \\ \mathsf{B}^{t}(i)\mathbb{P}(i) & 0 & -\gamma^2 I + L_f^{t}(i)L_f(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.47}$$

$$\begin{bmatrix} -\overline{\mathbb{L}}(i) & \mathbb{B}^{t}(i)L_f^{t}(i) & \mathbb{B}^{t}(i)\widehat{\mathbb{L}}(i) \\ L_f(i)\mathbb{B}(i) & -I & 0 \\ \widehat{\mathbb{L}}(i)\mathbb{B}(i) & 0 & -\widehat{\mathbb{L}}(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}. \tag{3.48}$$

Proof. Followed from Theorem 3.5 by setting $\overline{E}_A^{t} \equiv 0$, $\overline{E}_D^{t} \equiv 0$, $\overline{H}_A^{t} \equiv 0$. □

Having developed the basic analytical results in Theorems 3.5 and 3.6, we provide in the sequel expressions for the gain matrices of the observer (3.1) when applied to the neutral systems $(\Sigma_{\Delta A})$ and (Σ_A) while guaranteeing $\mathscr{H}_\infty$ performance in the light of Definition 3.1. For simplicity in exposition, we introduce the following matrix expressions for some scalars $\varepsilon > 0$, $\varrho > 0$:

$$\overline{A}_o(i) = \widehat{A}_o(i) + \gamma^{-2}N(i)N^{\mathrm{t}}(i)\mathbb{P}_x(i)(I + \Xi_c(i))^{-1},$$
$$\overline{C}_o(i) = \widehat{C}_o(i) + M(i)[\gamma^2 I - L^{\mathrm{t}}(i)L(i)]^{-1}N^{\mathrm{t}}(i)\mathbb{P}_x(i)(I + \Xi_c(i))^{-1}, \tag{3.49}$$

$$\begin{aligned}\Xi_{kk}(i) = {} & \big(\varepsilon^{-1}(i) + \varrho^{-1}(i)\big)H_c(i)H_c^{\mathrm{t}}(i) + \overline{C}_o(i)B(i)\Xi_{aa}^{-1}(i)B^{\mathrm{t}}(i)\widehat{C}_o^{\mathrm{t}}(i) \\ & + \big[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\big]\Xi_b^{-1}(i)\big[\overline{C}_o(i) + C_o(i)B(i) + C_d(i)\big]^{\mathrm{t}}, \\ \Xi_{gg}(i) = {} & \big(\varepsilon^{-1}(i) + \varrho^{-1}(i)\big)H_a(i)H_c^{\mathrm{t}}(i) \\ & + \Big(\overline{A}_o(i) + A_d(i) + \widehat{\mathbb{L}}_e(i)B(i)\Big)\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\overline{C}_o(i) \\ & + \Big(\overline{A}_o(i) - A_o(i) + \widehat{\mathbb{L}}_e(i)B(i)\Big)\Xi_b^{-1}(i)\big[\overline{C}_o(i) + C_o(i)B(i) + C_d(i)\big]^{\mathrm{t}}.\end{aligned} \tag{3.50}$$

The main results are summarized by the following theorems:

Theorem 3.7. *System $\Sigma_{\Delta A}$ is RSSID with a disturbance attenuation γ, if given matrices $0 < \mathbb{L}(i) = \mathbb{L}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$ and scalars $\xi(i) > 0$ such that*

$$\overline{\mathbb{L}}(i) = \mathbb{L}(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad \widehat{\mathbb{L}}(i) = \mathbb{L}(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}(m), \quad i \in \mathscr{S}$$

there exist matrices $0 < \mathbb{P}_e(i) = \mathbb{P}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{P}_x(i) = \mathbb{P}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $i \in \mathscr{S}$ and scalars $\varepsilon(i) > 0$, $\varrho(i) > 0$, $i \in \mathscr{S}$ satisfying the LMIs

$$\begin{bmatrix} \Upsilon_e(i) & \Lambda_b(i) & \Lambda_d(i) & \Lambda_f(i) & \mathbb{P}_x(i)N(i) \\ \Lambda_b^{\mathrm{t}}(i) & -\Phi_b(i) & 0 & 0 & 0 \\ \Lambda_d^{\mathrm{t}}(i) & 0 & -\Xi_a(i) & 0 & 0 \\ \Lambda_f^{\mathrm{t}}(i) & 0 & 0 & -\Xi_b(i) & 0 \\ N^{\mathrm{t}}(i)\mathbb{P}_x(i) & 0 & 0 & 0 & -\gamma^{-2}I + L^{\mathrm{t}}(i)L(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.51}$$

$$\begin{bmatrix} \Upsilon_x(i) & \Lambda_c(i) & \Lambda_h(i) & \mathbb{P}_x(i)N(i) \\ \Lambda_c^{\mathrm{t}}(i) & -\Phi_c(i) & 0 & 0 \\ \Lambda_h^{\mathrm{t}}(i) & 0 & -\Xi_b(i) & 0 \\ N^{\mathrm{t}}(i)\mathbb{P}_x(i) & 0 & 0 & -\gamma^{-2}I \end{bmatrix} < 0, \quad i \in \mathscr{S} \tag{3.52}$$

for all admissible uncertainties satisfying (2.8). Moreover, the estimator gains are given by

$$K_f(i) = \Xi_{gg}(i)\Xi_{kk}^{-1}(i), \qquad A_f(i) = \overline{A}_o(i) - K_f(i)\overline{C}_o(i). \tag{3.53}$$

Proof. Proceeding like Theorem 3.3, we express the expansion of (3.46) using (3.19) into the block form

$$\begin{bmatrix} \overline{\Sigma}_{11} & \overline{\Sigma}_{12} \\ \overline{\Sigma}_{12}^{\mathrm{t}} & \overline{\Sigma}_{22} \end{bmatrix}$$

where

$$\begin{aligned}\overline{\Sigma}_{11}(i) &= \mathbb{P}_e(i)A_f(i) + A_f^{\mathrm{t}}(i)\mathbb{P}_e(i) + \widehat{\mathbb{L}}_e(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}_e(m) + L^{\mathrm{t}}(i)L(i) \\ &\quad + (\varepsilon^{-1}(i) + \varrho^{-1}(i))\mathbb{P}_e(i)[H_a(i) - K_f(i)H_c(i)][H_a(i) - K_f(i)H_c(i)]^{\mathrm{t}}\mathbb{P}_e(i) \\ &\quad + \varepsilon^{-1}(i)\mathbb{P}_e(i)K_f(i)H_c(i)H_c^{\mathrm{t}}(i)K_f^{\mathrm{t}}(i)\mathbb{P}_e(i) \\ &\quad + \left[\mathbb{P}_e(i)[A_f(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_e(i)B(i)\right]\Xi_a^{-1}(i) \\ &\quad \cdot \left[\mathbb{P}_e(i)[A_f(i)B(i) + A_d(i)] + \widehat{\mathbb{L}}_e(i)B(i)\right]^{\mathrm{t}} \\ &\quad + \left[\mathbb{P}_e(i)[A_o(i) - A_f(i) - K_f(i)C_o(i)]B(i) - \mathbb{P}_e(i)K_f(i)C_d(i)\right]\Xi_b^{-1}(i) \\ &\quad \cdot \left[\mathbb{P}_e(i)[A_o(i) - A_f(i) - K_f(i)C_o(i)]B(i) - \mathbb{P}_e(i)K_f(i)C_d(i)\right]^{\mathrm{t}} \\ &\quad + \mathbb{P}_e(i)[N(i) - K_f(i)M(i)][\gamma^2 I - L^{\mathrm{t}}(i)L(i)]^{-1}[N(i) - K_f(i)M(i)]^{\mathrm{t}}(i)\mathbb{P}_x(i),\end{aligned} \tag{3.54}$$

$$\overline{\Sigma}_{22}(i) = \Sigma_{22}(i) + \gamma^{-2}\mathbb{P}_x(i)N(i)N^{\mathrm{t}}(i)\mathbb{P}_x(i), \tag{3.55}$$

$$\overline{\Sigma}_{12}(i) = \Sigma_{12}(i) + \mathbb{P}_e(i)[N(i) - K_f(i)M(i)][\gamma^2 I - L^{\mathrm{t}}(i)L(i)]^{-1}N^{\mathrm{t}}(i)\mathbb{P}_x(i). \tag{3.56}$$

Applying Fact 1 to the matrix block $\overline{\Sigma}_{22}$, we can readily obtain the LMI (3.52). The substitution of Eqs. (3.49) and (3.50) into (3.56) renders $\overline{\Sigma}_{12} \equiv 0$. Using (3.49), (3.50) and (3.53) into (3.54) with some matrix manipulations and applying the Schur complements we get the LMI (3.51). □

Theorem 3.8. *System Σ_A is SSID with a disturbance attenuation γ, if given matrices $0 < \mathbb{L}_e(i) = \mathbb{L}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}, 0 < \mathbb{L}_x(i) = \mathbb{L}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$ and scalars $\xi(i) > 0$ such that*

$$\overline{\mathbb{L}}_x(i) = \mathbb{L}_x(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad \widehat{\mathbb{L}}_x(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad i \in \mathscr{S},$$

$$\overline{\mathbb{L}}_e(i) = \mathbb{L}_e(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad \widehat{\mathbb{L}}_e(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad i \in \mathscr{S},$$

there exist matrices $0 < \mathbb{P}_e(i) = \mathbb{P}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{P}_x(i) = \mathbb{P}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $i \in \mathscr{S}$ satisfying the LMIs

$$\begin{bmatrix} \Upsilon_{eo}(i) & \Lambda_{do}(i) & \Lambda_{fo}(i) & \mathbb{P}_x(i)N(i) \\ \Lambda_{do}^{\mathrm{t}}(i) & -\Xi_a(i) & 0 & 0 \\ \Lambda_{fo}^{\mathrm{t}}(i) & 0 & -\Xi_{bo}(i) & 0 \\ N^{\mathrm{t}}(i)\mathbb{P}_x(i) & 0 & 0 & -\gamma^{-2}I + L^{\mathrm{t}}(i)L(i) \end{bmatrix} < 0, \quad i \in \mathscr{S}, \tag{3.57}$$

$$\begin{bmatrix} \Upsilon_{xo}(i) & \Lambda_h(i) & \mathbb{P}_x(i)N(i) \\ \Lambda_h^{\mathrm{t}}(i) & -\Xi_{bo}(i) & 0 \\ N^{\mathrm{t}}(i)\mathbb{P}_x(i) & 0 & -\gamma^{-2}I \end{bmatrix} < 0, \quad i \in \mathscr{S}. \tag{3.58}$$

Moreover, the estimator gains are given by

$$K_f(i) = \Xi_{ggo}(i)\Xi_{kko}^{-1}(i), \qquad A_f = \overline{A}_{oo} - K_f\overline{C}_{oo}. \tag{3.59}$$

Proof. By introducing

$$\begin{aligned} \overline{A}_{oo}(i) &= A_o(i) + \gamma^{-2}N(i)N^{\mathrm{t}}(i)\mathbb{P}_x(i)(I + \Xi_{co}(i))^{-1}, \\ \overline{C}_{oo}(i) &= \overline{C}_o + M(i)[\gamma^2 I - L^{\mathrm{t}}(i)L(i)]^{-1}N^{\mathrm{t}}(i)\mathbb{P}_x(I + \Xi_{co})^{-1}, \\ \Lambda_{fo}(i) &= \widehat{\mathbb{L}}_e(i)B(i), \\ \Xi_{kko} &= \overline{C}_oB\Xi_a^{-1}B^{\mathrm{t}}(i)\overline{C}_o^{\mathrm{t}}(i) + [[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)]\Xi_{bo}^{-1}(i) \\ &\quad \cdot \left[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\right]^{\mathrm{t}}, \\ \Xi_{ggo}(i) &= \left(\overline{A}_{oo}(i) + A_d(i) + \widehat{\mathbb{L}}_e(i)B(i)\right)\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\overline{C}_o(i) \\ &\quad + \left(\overline{A}_{oo}(i) - A_o(i) + \widehat{\mathbb{L}}_e(i)B(i)\right)\Xi_{bo}^{-1}(i)\left[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\right]^{\mathrm{t}}, \end{aligned} \tag{3.60}$$

and applying similar procedure to Theorem 3.3 while setting $\varrho \equiv 0$, $\varepsilon \equiv 0$, $H_a \equiv 0$, $E_a \equiv 0$, $H_c \equiv 0$, $E_d \equiv 0$ and using (3.56), the proof is completed. □

4. Robust stabilization

The foregoing theorems provided ways to produce a good replica of the state of the neutral system. Quite naturally, the next step would be to derive a robust state-estimate feedback control. For this purpose, we consider the following linear uncertain model:

$$\begin{aligned} (\Sigma_{\Delta nc}) \quad \mathscr{M}(\dot{x}_t) &\triangleq \dot{x}(t) - B(i)\dot{x}(t-\tau) \\ &= A_{\Delta o}(t,i)x(t) + A_{\Delta d}(t,i)x(t-\tau) \\ &\quad + F_{\Delta o}(t,i)u(t) + Nw(t), \end{aligned} \tag{4.1}$$

$$x(\eta) = \phi(\eta) \in \mathbb{C}([-\tau, 0], \mathbb{R}^n), \quad \forall \eta \in [-\tau, 0], \tag{4.2}$$

$$y(t) = C_{\Delta o}(t,i)x(t) + C_{\Delta d}(t,i)x(t-\tau) + M(i)w(t), \tag{4.3}$$

$$z(t) = L(i)x(t), \tag{4.4}$$

where $u(t) \in \mathbb{R}^q$ is the control input and

$$F_{\Delta o}(t,i) = F_o(i) + \Delta F_o(t,i) = F_o(i) + H_a(i)\Delta(t)E_b(i), \tag{4.5}$$

where $F_o(i) \in \mathbb{R}^{n\times q}$ and $E_b(i) \in \mathbb{R}^{\beta\times q}$ are known real matrices. The remaining matrices are as in Section 2. In the absence of uncertainties ($\Delta(\cdot) \equiv 0$), we obtain the following nominal neutral system:

$$(\Sigma_{nc}) \quad \mathscr{M}(\dot{x}_t) \triangleq \dot{x}(t) - B(i)\dot{x}(t-\tau) = A_o(i)x(t) + A_d(i)x(t-\tau) + F_o(i)u(t) + N(i)w(t), \tag{4.6}$$

$$x(\eta) = \phi(\eta) \in \mathbb{C}([-\tau,0],\mathbb{R}^n), \quad \forall\eta \in [-\tau,0], \tag{4.7}$$

$$y(t) = C_o(i)x(t) + C_d(i)x(t-\tau) + Mw(t), \tag{4.8}$$

$$z(t) = L(i)x(t). \tag{4.9}$$

In this section, we consider the problems of robust stabilization and stabilization of the neutral systems ($\Sigma_{\Delta nc}$) and (Σ_{nc}), respectively, using a linear memoryless state-estimate feedback control $u(t) = K_s(i)\hat{x}(t)$, $i \in \mathscr{S}$ where $\hat{x}(t)$ is generated by (3.1). It can be readily shown that the augmented system of ($\Sigma_{\Delta nc}$) and (Σ_e) takes the form:

$$(\Sigma_{\Delta AC}) \quad \mathscr{M}(\dot{\zeta}_t) \triangleq \dot{\zeta}(t) - \mathbb{B}(i)\dot{\zeta}(t-\tau) = \mathscr{A}_{\Delta c}(t,i)\zeta(t) + \mathscr{D}_{\Delta o}(t,i)\zeta(t-\tau) + \mathsf{B}(i)w(t), \tag{4.10}$$

$$\tilde{z}(t) = L_f(i)\zeta(t), \tag{4.11}$$

where

$$\mathscr{A}_{\Delta c}(t,i) = [\mathsf{A}_c(i) + \overline{H}_A(i)\Delta(t)\overline{E}_{AC}(i)], \qquad \overline{E}_{AC}(i) = [0 \quad E_a(i) + E_b(i)K_s(i)], \tag{4.12}$$

$$\mathsf{A}_c(i) = \begin{bmatrix} A_f(i) - F_o(i)K_s(i) & A_o(i) + F_o(i)K_s(i) - K_f(i)C_o(i) - A_f(i) \\ 0 & A_o(i) \end{bmatrix}. \tag{4.13}$$

The remaining matrices are given by (3.7)–(3.9). It follows from Theorem 3.1 that ($\Sigma_{\Delta AC}$) is RSSID if for given matrices $0 < \mathbb{L}(i) = \mathbb{L}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$, there exist matrices $0 < \mathbb{P}(i) = \mathbb{P}^{\mathrm{t}}(i) \in \mathbb{R}^{2n\times 2n}$, $i \in \mathscr{S}$ and scalars $\xi(i) > 0$, $\varepsilon(i) > 0$, $\varrho(i) > 0$, $i \in \mathscr{S}$ such that the following inequality holds:

$$\mathbb{P}(i)\mathsf{A}_c(i) + \mathsf{A}_c^{\mathrm{t}}(i)\mathbb{P}(i) + \widehat{\mathbb{L}}(i) + \sum_{m=1}^{s}\alpha_{im}\mathbb{P}(m) + \varrho^{-1}(i)\mathbb{P}(i)\overline{H}_A(i)\overline{H}_A^{\mathrm{t}}(i)\mathbb{P}(i)$$
$$+ \varrho(i)\overline{E}_{AC}^{\mathrm{t}}(i)\overline{E}_{AC}(i) + \varepsilon^{-1}(i)\mathbb{P}(i)\left[\overline{H}_A(i)\overline{H}_A^{\mathrm{t}}(i) + \overline{H}_D(i)\overline{H}_D^{\mathrm{t}}(i)\right]\mathbb{P}(i)$$

$$+\left(\mathbb{P}(i)[\mathsf{A}_c(i)\mathbb{B}(i)+\mathsf{D}(i)]+\widehat{\mathbb{L}}(i)\mathbb{B}(i)\right)\Big[\overline{\mathbb{L}}-\mathbb{B}^{\mathrm{t}}(i)\widehat{\mathbb{L}}(i)\mathbb{B}(i)$$
$$-\varepsilon(i)\left(\overline{E}_A^{\mathrm{t}}(i)\overline{E}_A(i)+\overline{E}_D^{\mathrm{t}}(i)\overline{E}_D(i)\right)\Big]^{-1}\left(\mathbb{P}(i)[\mathsf{A}_c(i)\mathbb{B}(i)+\mathsf{D}(i)]+\widehat{\mathbb{L}}(i)\mathbb{B}(i)\right)^{\mathrm{t}}. \tag{4.14}$$

Taking into account (3.7)–(3.9), (3.26)–(3.28) and (4.13), we express (4.14) into the block form

$$\begin{bmatrix} \Omega_{11}(i) & \Omega_{12}(i) \\ \Omega_{12}^{\mathrm{t}}(i) & \Omega_{22}(i) \end{bmatrix},$$

where

$$\begin{aligned}\Omega_{11}(i) = {} & \mathbb{P}_e(i)[A_f(i)-F_o(i)K_s(i)]+[A_f(i)-F_o(i)K_s(i)]^{\mathrm{t}}\mathbb{P}_e(i)+\widehat{\mathbb{L}}_e(i)\\ & +\varepsilon^{-1}(i)\mathbb{P}_e(i)K_f(i)H_c(i)H_c^{\mathrm{t}}(i)K_f^{\mathrm{t}}(i)\mathbb{P}_e(i)+\sum_{m=1}^{s}\alpha_{im}\mathbb{P}_e(m)\\ & +(\varepsilon^{-1}(i)+\varrho^{-1}(i))\mathbb{P}_e(i)[H_a(i)-K_f(i)H_c(i)][H_a(i)\\ & -K_f(i)H_c(i)]^{\mathrm{t}}\mathbb{P}_e(i)+\Big[\mathbb{P}_e(i)[A_f(i)B(i)+A_d(i)]+\widehat{\mathbb{L}}_e(i)B(i)\Big]\Xi_a^{-1}(i)\\ & \times\Big[\mathbb{P}_e(i)[A_f(i)B(i)+A_d(i)]+\widehat{\mathbb{L}}_e(i)B(i)\Big]^{\mathrm{t}}\\ & +\big[\mathbb{P}_e(i)[A_o(i)+F_o(i)K_s(i)-A_f(i)-K_f(i)C_o(i)]B(i)\\ & -\mathbb{P}_e(i)K_f(i)C_d(i)\big]\Xi_w^{-1}(i)\big[\mathbb{P}_e(i)[A_o(i)+F_o(i)K_s(i)-A_f(i)\\ & -K_f(i)C_o(i)]B(i)-\mathbb{P}_e(i)K_f(i)C_d(i)\big]^{\mathrm{t}},\end{aligned} \tag{4.15}$$

$$\begin{aligned}\Omega_{22}(i) = {} & \mathbb{P}_x(i)A_o(i)+A_o^{\mathrm{t}}(i)\mathbb{P}_x(i)+\widehat{\mathbb{L}}_x(i)\\ & +\sum_{m=1}^{s}\alpha_{im}\mathbb{P}_x(m)+\varrho(i)[E_a(i)+E_b(i)K_s(i)]^{\mathrm{t}}(i)\Big[E_a(i)\\ & +E_b(i)K_s(i)](i)+(\varepsilon^{-1}(i)+\varrho^{-1}(i))\mathbb{P}_x(i)H_a(i)H_a^{\mathrm{t}}(i)\mathbb{P}_x(i)\\ & +\varepsilon^{-1}(i)\mathbb{P}_x(i)H_c(i)H_c^{\mathrm{t}}(i)\mathbb{P}_x(i)+[\mathbb{P}_x(i)[A_o(i)B(i)+A_d(i)]\\ & +\widehat{\mathbb{L}}_x(i)B(i)\Big]\Xi_b^{-1}(i)\Big[\mathbb{P}_x(i)[A_o(i)B(i)+A_d(i)]+\widehat{\mathbb{L}}_x(i)B(i)\Big]^{\mathrm{t}},\end{aligned} \tag{4.16}$$

$$\begin{aligned}\Omega_{12}(i) = {} & \big[\mathbb{P}_e(i)[A_o(i)+F_o(i)K_s(i)-A_f(i)-K_f(i)C_o(i)]B(i)\\ & -\mathbb{P}_e(i)K_f(i)C_d(i)\big]\Xi_b^{-1}(i)\Big[\mathbb{P}_x(i)[A_o(i)B(i)+A_d(i)]+\widehat{\mathbb{L}}_x(i)B(i)\Big]^{\mathrm{t}}\\ & -\varepsilon^{-1}\mathbb{P}_e(i)K_f(i)H_c(i)H_c^{\mathrm{t}}(i)K_f^{\mathrm{t}}(i)\mathbb{P}_x(i)+\mathbb{P}_e(i)[A_o(i)+F_o(i)K_s(i)\\ & -A_f(i)-K_f(i)C_o(i)]+(\varepsilon^{-1}(i)+\varrho^{-1}(i))\mathbb{P}_e(i)\\ & \times[H_a(i)-K_f(i)H_c(i)]H_a^{\mathrm{t}}(i)\mathbb{P}_x(i),\end{aligned} \tag{4.17}$$

where

$$\begin{aligned}\Xi_w(i) &= \overline{\mathbb{L}}_x(i) - B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i)B(i) \\ &\quad - \varepsilon(i)\big([E_a(i)+E_b(i)K_s(i)]^{\mathrm{t}}[E_a(i)+E_b(i)K_s(i)] + E_d^{\mathrm{t}}(i)E_d(i)\big).\end{aligned} \tag{4.18}$$

The main robust stabilization result is summarized by the following theorem:

Theorem 4.1. *System* $(\Sigma_{\Delta AC})$ *is RSSID via memoryless state-feedback* $u(t) = K_s(i)x(t)$, $i \in \mathscr{S}$, *if given matrices* $0 < \mathbb{L}_e(i) = \mathbb{L}_e^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{L}_x(i) = \mathbb{L}_x^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$ *and scalars* $\xi(i) > 0$ *such that*

$$\overline{\mathbb{L}}_x(i) = \mathbb{L}_x(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad \widehat{\mathbb{L}}_x(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_x(m), \quad i \in \mathscr{S},$$

$$\overline{\mathbb{L}}_e(i) = \mathbb{L}_e(i) - \xi^{-1}(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad \widehat{\mathbb{L}}_e(i) = \mathbb{L}_x(i) + \xi(i)\sum_{m=1}^{s}\alpha_{im}\mathbb{L}_e(m), \quad i \in \mathscr{S},$$

there exist matrices $0 < Y(i) = Y^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < X(i) = X^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < Z(i) = Z^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $W(i) \in \mathbb{R}^{q\times n}$, $i \in \mathscr{S}$ *and scalars* $\xi(i) > 0$, $\varepsilon(i) > 0$, $\varrho(i) > 0$, $i \in \mathscr{S}$ *satisfying the LMIs*

$$\begin{bmatrix} \Gamma_{ek}(i) & \Lambda_b(i) & \Lambda_m(i) & \Lambda_q(i) \\ \Lambda_b^{\mathrm{t}}(i) & -\Phi_b(i) & 0 & 0 \\ \Lambda_m^{\mathrm{t}}(i) & 0 & -\Xi_a(i) & 0 \\ \Lambda_q^{\mathrm{t}}(i) & 0 & 0 & -\Xi_b(i) \end{bmatrix} < 0, \tag{4.19}$$

$$\begin{bmatrix} \Upsilon_{xk}(i) & \Lambda_c(i) & \Lambda_{vx}(i) \\ \Lambda_c^{\mathrm{t}}(i) & -\Phi_c(i) & 0 \\ \Lambda_{vx}^{\mathrm{t}}(i) & 0 & -\Xi_b(i) \end{bmatrix} < 0, \qquad \begin{bmatrix} Y(i) & I \\ I & Z(i) \end{bmatrix} \geqslant 0, \tag{4.20}$$

where

$$\begin{aligned}\Xi_{gs}(i) &= \Xi_g(i) + F_o(i)W(i)Z(i)\Big\{\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\widehat{C}_o(i) + \Xi_b^{-1}(i) \\ &\qquad \times[[\widehat{C}_o(i) + C_o(i)]B(i) + C_d(i)]^{\mathrm{t}}\Big\}, \\ \Gamma_{ek}(i) &= \widehat{A}_o(i)Y(i) + Y(i)\widehat{A}_o^{\mathrm{t}}(i) + Y(i)\widehat{\mathbb{L}}_e(i)Y(i) - \Xi_{gs}^{\mathrm{t}}(i)\Xi_k^{-1}(i)\Xi_{gs}(i) \\ &\qquad + F_o(i)W(i) + W^{\mathrm{t}}(i)F_o^{\mathrm{t}}(i) \\ \Lambda_m(i) &= \widehat{A}_o(i) + A_d(i) + Y(i)\widehat{\mathbb{L}}_e(i)B(i) + F_o(i)W(i)Z(i) + Z(i)W^{\mathrm{t}}(i)F_o^{\mathrm{t}}(i), \\ \Lambda_q(i) &= \widehat{A}_o(i) - A_o(i) + Y(i)\widehat{\mathbb{L}}_e(i)B(i) + F_o(i)W(i)Z(i) + Z(i)W^{\mathrm{t}}(i)F_o^{\mathrm{t}}(i),\end{aligned}$$

$$\Upsilon_{xk}(i) = X(i)A_o(i) + A_o^{\mathrm{t}}(i)X(i) + \widehat{\mathbb{L}}_x(i) + \varrho(i)[E_a(i) + E_b(i)W(i)Z(i)]^{\mathrm{t}}[E_a(i) + E_b(i)W(i)Z(i)],$$

$$\Xi_s(i) = B(i)\Xi_b^{-1}(i)\left[B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i) + [A_d^{\mathrm{t}}(i) + B^{\mathrm{t}}(i)A_o^{\mathrm{t}}(i)]X(i)\right],$$

$$\Lambda_{vx}(i) = X(i)A_o(i)B(i) + X(i)A_d(i) + \widehat{\mathbb{L}}_x(i)B(i),$$

$$\widetilde{A}_o(i) = A_o(i) + H_a(i)H_a^{\mathrm{t}}(i)X(i)(I + \Xi_s(i))^{-1} \tag{4.21}$$

for all admissible uncertainties satisfying (2.8). Moreover, the associated gains are given by

$$K_f(i) = \Xi_{gs}(i)\Xi_k^{-1}(i),$$
$$A_f(i) = \widetilde{A}_o(i) + F_o(i)W(i)Z(i) - K_f(i)\widehat{C}_o(i), \tag{4.22}$$
$$K_s(i) = W(i)Z(i).$$

Proof. By defining

$$\mathbb{P}_x(i) = X(i), \qquad \mathbb{P}_e^{-1}(i) = Y(i), \qquad K_s(i) = W(i)Z(i)$$

and applying the technique of Theorem 3.3 using (3.23), (3.24) and (4.22), the desired result is readily obtained. □

On the other hand, by combining the nominal system of (Σ_{nc}) and (Σ_e) we obtain the augmented system:

$$(\Sigma_{AC}) \quad \mathscr{M}(\dot{\zeta}_t) \triangleq \dot{\zeta}(t) - \mathbb{B}\dot{\zeta}(t-\tau) = \mathsf{A}_c\zeta(t) + \mathsf{D}\zeta(t-\tau) + \mathsf{B}w(t), \tag{4.23}$$

$$\tilde{z}(t) = L_f\zeta(t) \tag{4.24}$$

and for which we prove the following theorem.

Theorem 4.2. *System (Σ_{AC}) is SSID via memoryless state-feedback $u(t) = K_s(i)x(t)$, $i \in \mathscr{S}$, if there exist matrices $0 < \mathbb{L}_e(i) \in \mathbb{R}^{n\times n}$, $0 < \mathbb{L}_x(i) \in \mathbb{R}^{n\times n}$, $0 < Y(i) = Y^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < X(i) = X^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $0 < Z(i) = Z^{\mathrm{t}}(i) \in \mathbb{R}^{n\times n}$, $W(i) \in \mathbb{R}^{q\times n}$, $i \in \mathscr{S}$ satisfying the LMIs*

$$\begin{bmatrix} \Gamma_{eko}(i) & \Lambda_{mo}(i) & \Lambda_{qo}(i) \\ \Lambda_{mo}^{\mathrm{t}}(i) & -\Xi_a(i) & 0 \\ \Lambda_{qo}^{\mathrm{t}}(i) & 0 & -\Xi_{bo}(i) \end{bmatrix} < 0, \qquad \begin{bmatrix} \Upsilon_{xko}(i) & \Lambda_{vx}(i) \\ \Lambda_{vx}^{\mathrm{t}}(i) & -\Xi_{bo}(i) \end{bmatrix} < 0, \tag{4.25}$$

$$\begin{bmatrix} Y(i) & I \\ I & Z(i) \end{bmatrix} \geqslant 0, \tag{4.26}$$

where

$$\begin{aligned}
\Xi_{gso} &= \Xi_{go}(i) + F_o(i)W(i)Z(i)\Big\{\Xi_a^{-1}(i)B^{\mathrm{t}}(i)\overline{C}_o(i) \\
&\quad + \Xi_b^{-1}(i)\big[[\overline{C}_o(i) + C_o(i)]B(i) + C_d(i)\big]^{\mathrm{t}}\Big\} \\
\Gamma_{eko}(i) &= A_o(i)Y(i) + Y(i)A_o^{\mathrm{t}}(i) + Y(i)\widehat{\mathbb{L}}_e(i)Y(i) - \Xi_{gso}^{\mathrm{t}}(i)\Xi_{ko}^{-1}(i)\Xi_{gso}(i) \\
&\quad + F_o(i)W(i) + W^{\mathrm{t}}(i)F_o^{\mathrm{t}}(i), \\
\Lambda_{mo}(i) &= A_o(i) + A_d(i) + Y(i)\widehat{\mathbb{L}}_e(i)B(i) + F_o(i)W(i)Z(i) + Z(i)W^{\mathrm{t}}(i)F_o^{\mathrm{t}}(i), \\
\Lambda_{qo}(i) &= Y(i)\widehat{\mathbb{L}}_e(i)B(i) + F_o(i)W(i)Z(i) + Z(i)W^{\mathrm{t}}(i)F_o^{\mathrm{t}}(i), \\
\Upsilon_{xko}(i) &= X(i)A_o(i) + A_o^{\mathrm{t}}(i)X(i) + \widehat{\mathbb{L}}_x(i), \\
\Xi_s(i) &= B(i)\Xi_b^{-1}(i)\Big[B^{\mathrm{t}}(i)\widehat{\mathbb{L}}_x(i) + [A_d^{\mathrm{t}}(i) + B^{\mathrm{t}}(i)A_o^{\mathrm{t}}(i)]X(i)\Big], \\
\Lambda_{vx}(i) &= X(i)A_o(i)B(i) + X(i)A_d(i) + \widehat{\mathbb{L}}_x(i)B(i), \\
\widetilde{A}_o(i) &= A_o(i). \qquad (4.27)
\end{aligned}$$

Moreover, the associated gains are given by

$$\begin{aligned}
K_f(i) &= \Xi_{gso}(i)\Xi_{ko}^{-1}(i), \\
A_f(i) &= A_o(i) + F_o(i)W(i)Z(i) - K_f(i)\overline{C}_o(i), \qquad (4.28) \\
K_s(i) &= W(i)Z(i).
\end{aligned}$$

Proof. Follows from Theorem 4.1 in the manner theorem by suppressing the uncertain terms. □

5. Examples

In order to illustrate the theoretical results of this paper, we provide some numerical examples.

5.1. Example 1

We consider a pilot-scale single-reach water quality system which can fall into the type (2.3)–(2.5) with $\tau = 0.75$. Let the Markov process governing the mode switching has generator

$$\mathfrak{I} = \begin{bmatrix} -4 & 4 \\ 3 & -3 \end{bmatrix}.$$

For the two operating conditions (modes), the associated date are:

Mode 1:

$$A_o(1) = \begin{bmatrix} 0 & 2 \\ -3 & -6 \end{bmatrix}, \quad A_d(1) = \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.3 \end{bmatrix}, \quad C_o(1) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix},$$

$$C_d(1) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix},$$

$$B(1) = \begin{bmatrix} 0.15 & 0, \\ 0 & 0.15 \end{bmatrix}, \quad N(1) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad M(1) = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad H_a(1) = \begin{bmatrix} 0.1 \\ 0 \end{bmatrix},$$

$$L(1) = [0.1 \quad 0.1], \quad E_a(1) = [0.1 \quad 0.2], \quad E_d(1) = [0.1 \quad 0].$$

Mode 2:

$$A_o(2) = \begin{bmatrix} 0 & 2 \\ -3 & -4 \end{bmatrix}, \quad A_d(2) = \begin{bmatrix} -0.1 & 0 \\ 0.2 & -0.4 \end{bmatrix}, \quad C_o(2) = \begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix},$$

$$C_d(2) = \begin{bmatrix} 0.2 & 0 \\ 0 & 0.2 \end{bmatrix},$$

$$B(2) = \begin{bmatrix} 0.1 & 0 \\ 0 & 0.1 \end{bmatrix}, \quad N(2) = \begin{bmatrix} 0.2 \\ 0.1 \end{bmatrix}, \quad M(2) = \begin{bmatrix} 0.1 \\ 0.2 \end{bmatrix}, \quad H_a(2) = \begin{bmatrix} 0 \\ 0.2 \end{bmatrix},$$

$$L(2) = [0.1 \quad 0.1], \quad E_a(2) = [0.2 \quad 0.1], \quad E_d(2) = [0 \quad 0.3].$$

First we note that Assumptions 2.1 and 2.2 are met for both modes. Invoking the software environment [10], we solve the LMIs (3.31) using (3.27)–(3.33) and the initial data for $i = 1, 2$:

$$\mathbb{L}_e(1) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \quad \mathbb{L}_x(1) = \begin{bmatrix} 5 & 0 \\ 0 & 5 \end{bmatrix}, \quad \xi(1) = 20,$$

$$\mathbb{L}_e(2) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad \mathbb{L}_x(2) = \begin{bmatrix} 4 & 0 \\ 0 & 4 \end{bmatrix}, \quad \xi(2) = 25,$$

which ensures that $\overline{\mathbb{L}}_e(1) > 0$, $\widehat{\mathbb{L}}_e(1) > 0$, $\overline{\mathbb{L}}_x(2) > 0$, $\widehat{\mathbb{L}}_x(2) > 0$. The feasible solutions are given by

$$\mathbb{P}_e(1) = \begin{bmatrix} 1.8010 & 0.0577 \\ 0.0577 & 1.3405 \end{bmatrix}, \quad \mathbb{L}_e(1) = \begin{bmatrix} 0.7413 & 0.0029 \\ 0.0029 & 0.9746 \end{bmatrix},$$

$$\mathbb{P}_x(1) = \begin{bmatrix} 1.0134 & 0.0106 \\ 0.0106 & 0.8245 \end{bmatrix},$$

$$\mathbb{L}_x(1) = \begin{bmatrix} 0.8350 & 0.0217 \\ 0.0217 & 1.1205 \end{bmatrix}, \quad \varrho(1) = 2.7694,$$

$$\varepsilon(1) = 6.1246, \quad \gamma = 1.1045,$$

$$K_f(1) = \begin{bmatrix} 0.0404 & -2.6255 \\ 1.0095 & -3.1423 \end{bmatrix}, \quad A_f(1) = \begin{bmatrix} 0.0322 & 1.3146 \\ -2.3011 & -5.8945 \end{bmatrix},$$

$$\mathbb{P}_e(2) = \begin{bmatrix} 2.9627 & 0.1527 \\ 0.1527 & 0.8405 \end{bmatrix}, \quad \mathbb{L}_e(2) = \begin{bmatrix} 1.1174 & 0.0109 \\ 0.0109 & 1.1657 \end{bmatrix},$$

$$\mathbb{P}_x(2) = \begin{bmatrix} 0.9467 & 0.0105 \\ 0.0105 & 1.18542 \end{bmatrix},$$

$$\mathbb{L}_x(2) = \begin{bmatrix} 0.8350 & 0.0217 \\ 0.0217 & 1.1205 \end{bmatrix}, \quad \varrho(2) = 3.2658, \quad \varepsilon(2) = 4.8453,$$

$$K_f(2) = \begin{bmatrix} 0.1107 & -2.9167 \\ 1.1945 & -4.4627 \end{bmatrix}, \quad A_f(2) = \begin{bmatrix} 0.0176 & 1.5031 \\ -3.0111 & -3.9213 \end{bmatrix}.$$

This verifies Theorem 3.7 and in turn confirms the robust stochastic stability independent of delay and with disturbance attenuation $\gamma = 1.1045$ of the water quality model.

5.2. Example 2

To illustrate Theorem 4.1, we consider the numerical data of Example 1 in addition to

$$F_o(1) = \begin{bmatrix} 1 & 0 \\ 0 & 2 \end{bmatrix}, \; F_o(2) = \begin{bmatrix} 2 & 0 \\ 0 & 1 \end{bmatrix}, \; E_b(1) = [0.1 \;\; 0.1], \; E_b(2) = [0.2 \;\; 0.2]$$

and rely again on the software package [10]. Here, we solve the LMIs. (4.19), (4.20) using (4.21) for $i = 1, 2$ to produce the feasible solutions:

$$X(1) = \begin{bmatrix} 2.3506 & 0.9457 \\ 0.9457 & 2.5294 \end{bmatrix}, \quad Y(1) = \begin{bmatrix} 1.5980 & 0.2654 \\ 0.2654 & 1.4533 \end{bmatrix},$$

$$Z(1) = \begin{bmatrix} 1.5512 & -0.2055 \\ -0.2055 & 0.8733 \end{bmatrix},$$

$$W(1) = \begin{bmatrix} -0.7524 & -0.1445 \\ 0.1281 & 1.2206 \end{bmatrix}, \quad \varrho(1) = 3.1338,$$

$$\varepsilon(1) = 8.1457, \quad \gamma = 1.2671,$$

$$K_f(1) = \begin{bmatrix} 0.0404 & -2.6255 \\ 1.0095 & -3.1423 \end{bmatrix}, \quad A_f(1) = \begin{bmatrix} 0.0322 & 1.3146 \\ -2.3011 & -5.8945 \end{bmatrix},$$

$$K_s(1) = \begin{bmatrix} -1.1374 & 0.284 \\ -0.0521 & 1.0396 \end{bmatrix},$$

$$Y(2) = \begin{bmatrix} 2.9627 & 0.1527 \\ 0.1527 & 0.8405 \end{bmatrix}, \quad X(2) = \begin{bmatrix} 1.1174 & 0.0109 \\ 0.0109 & 1.1657 \end{bmatrix},$$
$$Z(2) = \begin{bmatrix} 1.1054 & -0.2104 \\ -0.2104 & 1.2132 \end{bmatrix},$$
$$W(2) = \begin{bmatrix} 0.9450 & -0.5117 \\ 0.4402 & -1.4204 \end{bmatrix}, \quad \varrho(2) = 3.4116, \quad \varepsilon(2) = 5.5514,$$
$$K_f(2) = \begin{bmatrix} 0.1107 & -2.9167 \\ 1.1945 & -4.4627 \end{bmatrix}, \quad A_f(2) = \begin{bmatrix} 0.0176 & 1.5031 \\ -3.0111 & -3.9213 \end{bmatrix},$$
$$K_s(2) = \begin{bmatrix} 1.0578 & -0.8196 \\ 0.7414 & -1.8158 \end{bmatrix}.$$

6. Conclusions

In this paper, the designs of robust observation, robust $\mathscr{H}_\infty$ observation and robust stabilization methods for a class of linear neutral-type continuous-time systems with norm-bounded parametric uncertainties have been presented. A linear state-delayed estimator is proposed such that the augmented system achieves desirable stability properties independent of delay. Then a memoryless state-estimate feedback control to stabilize the closed-loop system is designed. In all cases, the gain matrices are determined by solving linear matrix inequalities with scaling parameters. Two numerical examples are included to illustrate the validity of the theoretical results.

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