Stability Analysis of Network Data Flow Control for Dynamic Link Capacity Case

Yuequan Yang[1], Yaqin Li[1], Min Tan[2], Jianqiang Yi[2],
John T. Wen[3], and Xuewu Guo[4]

[1] College of Information Engineering, Yangzhou University,
Yangzhou 225009, China
[2] Key Laboratory of Complex Systems and Intelligence Science,
Institute of Automation, Chinese Academy of Sciences,
Beijing 100080, China
[3] Department of Electrical, Computer and Systems Engineering,
Rensselaer Polytechnic Institute, Troy, NY 12180, USA
[4] Shanxi Jinyao Coking Co. Ltd, Shanxi 030001, China
yangyq@yzu.edu.cn, yyquan@compsys.ia.ac.cn

Abstract. Much attention has been paid upon network data flow control in recent years. The main problem in this field is how to design good algorithm or control law for flow rate of network data flow sources and for updated price of communication links. Base on Lyapunov stability theory, this paper makes a deep analysis of stability of such network data flow control systems with the consideration of dynamic link capacity case. Simulations show that the stability analysis of dynamic link capacity of network data flow control system provided in the paper is enlightening and meaningful to further understand and design good control strategy of network data flow control problem.

1 Introduction

In recent years congestion control has attracted much interest in the field of control theory research. Many good regulation methods and control schemes have been proposed. In the Internet environment, network flow is governed by the interconnection between information sources and communication links [1-9]. With this view, the central problem is to seek good regulation law for each source rate and update law of price for communication links. F. Kelly, A. Maulloo, and D. Tan (1988, [1]) and S. H. Low and D. E. Lapsley (1999, [2]) provided a common approach to flow control, that is, to decompose the problem into a static optimization problem and a dynamic stabilization problem. For the optimization the main task is to design algorithms to approximate to equilibrium of the closed loop system with some constraints of available queue length or available link rates based on gradient projection optimization techniques. In [2], the synchronous algorithm and asynchronous update algorithm are proposed. S. H. Low, F. Paganini, and J. C. Doyle (2002, [3]) pointed out that congestion control mechanisms in today's Internet already represent one of the largest deployed artificial feedback systems. In [3], comprehensive

D.-S. Huang, K. Li, and G.W. Irwin (Eds.): ICIC 2006, LNCIS 344, pp. 395 – 406, 2006.

description and analysis were given with optimization-based framework. Considering the presence of communication delays, R. Johari and D. K. H. Tan (2001, [4]) gave stability condition of network rate control for both a single resource and a large network with constant round-trip delay.

Especially, J. T. Wen (2004, [5]) developed a unifying framework for stabilizing source and link control laws, which encompass many existing algorithms in [1,6] and many special cases. Based on passivity theory, J. T. Wen proposed a comprehensive equilibrium stability analysis and dynamic control law design by constructing different passive system or strictly passive systems instead. The greatest advantage of this novel strategy is the combination of equilibrium stability and dynamic control law design for network flow control. Though, because the complexity and variety of network environment, capacity of links to be allocated is not static, but often dynamic according to its available bit rate for some networks such as ATM network environment in reality. Contraposing to this problem, this paper mainly aims at giving deep stability analysis of the optimum equilibrium manifolds of the primal optimal problem and its dual problem based on some mild conditions.

This paper is organized as follows. In section 2, we provide some preliminary knowledge including some critical concepts, such as positive projection, strictly passivity and other critical results to be used in next sections. The main problem under consideration is described in detail in section 3. The deep L_p stability analysis of the two kinds of systems based on rate control for information sources and link price update price law models is given based on Lyapunov stability theory in section 4. The simulation experiments are done in section 5. Finally, conclusion is made in section 6.

2 Preliminary Knowledge

In this section, some preliminary knowledge about passivity theory and some critical results are given as follows.

Definition 2.1. [5] Positive projection $(f(x))_x^+$ with some function $f(x)$ is defined as follows

$$(f(x))_x^+ = \begin{cases} f(x) & , \text{ if } x > 0, \text{or } x = 0 \text{ and } f(x) \geq 0 \\ 0 & , \quad \text{if } x = 0 \text{ and } f(x) < 0 \end{cases} \tag{1}$$

Now, assume there exist a system H , in which system state is vector $x \in R^N$, input $u \in R^M$ and output $y \in R^M$. According to passivity theory[10], some definitions are given as follows.

Definition 2.2. The system H above is called passivity, if there exists a continuously differentiable energy function $V(x) \geq 0$ satisfying $\dot{V}(x) \leq -W(x) + u^T y$ for some $W(x) \geq 0$.

From the above definition, some system is passive if the system itself doesn't generate energy, but possibly dissipates the energy injected into the system.

Definition 2.3. The system H above is called strictly passivity, if there exists a continuously differentiable energy function $V(x) \geq 0$ satisfying $\dot{V}(x) \leq -W(x) + u^T y$ for some $W(x) > 0$.

Definition 2.4. The system is L_p stable, if L_p -norm of the state vector, and output vector of the controlled system exist simultaneously for $p > 1$, or $p = \infty$ if input vector variable is L_p.

Lemma 2.1[11]**.** Suppose that $W : [0, \infty) \to -\alpha W(t) + \beta(t)$ satisfies

$$D^+ W(t) \leq -\alpha W(t) + \beta(t) \tag{2}$$

where D^+ denotes the upper Dini derivative, α is a positive constant, and $\beta \in L_p, p \in [1, \infty)$, then

$$\|W\|_{L_p} \leq (\alpha p)^{-\frac{1}{p}} W(0) + (\alpha q)^{\frac{1}{q}} \|\beta\|_{L_p} \tag{3}$$

where p, q satisfies

$$\frac{1}{p} + \frac{1}{q} = 1 \tag{4}$$

and when $p = \infty$, the following estimate holds

$$\|W\| \leq e^{-\alpha t} \|W(0)\| + \alpha^{-1} \|\beta\|_{L_\infty} \tag{5}$$

3 Problem Formulation

As we know, status of network environment is often changing along with time. Not only the number of network data flow source but also link rate capacity/bandwidth is variable. In fact, some network links in certain routing path might fail in work. In this note, we don't plan to make consideration of this problem. Now we focus upon such an occasion that link rate capacities for sources are not constant, but dynamic changing along with time, denoted as $c(t)$ with respect to time variable t. Firstly, we assume network is constructed by N information resource and L communication links where N and L are some known integer number. Now we define a routing matrix $R = (r_{ij})_{L \times N}$ with

$$r_{ij} = \begin{cases} 1, & \text{if the source } j \text{ passes through the link } i \\ 0, & \text{other} \end{cases} \tag{6}$$

From the routine matrix, it is easily to define two sets $s(l)$ and $l(s)$ respectively, that is, $s(l)=\{j \mid r_{lj}=1 \;, j=1,\cdots,N\}$ and $l(s)=\{i \mid r_{is}=1 \; j=1,\cdots,L\}$. Then the aggregate rate vector of links $y\in R^L$ and aggregate price of sources $q\in R^N$ can be defined respectively as

$$y=Rx, \text{ and } q=R^T p \tag{7}$$

where $x\in R^N$ is called source rate vector and $p\in R^L$ link price vector.

In [1], the information flow control problem is described a static optimization prime problem and its dual problem as follows

$$\max_{x\geq 0} \sum_{i=1,\cdots,N} U_i(x_i) \qquad \text{s. t. } Rx \leq c(t) \tag{8}$$

where $U_i(x_i)$ is utilization function with the strictly concave property and $c(t)\in R^L$ is link rate capacity vector with component $c_l(t)$ representing the rate capacity of the link l. Its dual problem is easily obtained as follows

$$\min_{p\geq 0} \max_{x\geq 0} \sum_{i=1,\cdots,N} U_i(x_i) - \sum_{l=1,\cdots,L} p_l(y_l - c_l(t)) . \tag{9}$$

After simple transformation, the above can be converted to be

$$\min_{p\geq 0} \max_{x\geq 0} \sum_{i=1,\cdots,N} \{U_i(x_i) - q_i x_i\} + \sum_{l=1,\cdots,L} p_l c_l(t) \tag{10}$$

If $U(x)$ is differentiable, the first order condition for the maximization problem is

$$U_i^{'}(x_i) = q_i \tag{11}$$

and

$$p_l \begin{cases} =0, & \text{if } y_l < c_l(t) \\ \geq 0, & \text{if } y_l = c_l(t) \end{cases} \tag{12}$$

where $U_i^{'}(x_i)=\partial U_i/\partial x_i$. From appendix I in [5], we know that if $U_i(x_i)$ is strictly concave and routing matrix R is full row rank the (11) and (12) are sufficient to determine unique equilibrium for the constant link capacity. But what will happen when link capacity becomes dynamic changing along with time? The objective of flow rates and link update laws is to drive the actual source rate and link prices to their respective equilibrium dynamic. To realize this destination, there are several real constraints exist, such as decentralization network topology, no routing information, that is, routing matrix is unknown to network data flow source, no coordination and the link capacity is time varying.

In [1], the source update law is given by

$$\dot{x} = K(U'(x) - q(t))^{+}_{x} \tag{13}$$

where $K = diag\{k\}$ $K = diag\{k_i\}, k_i > 0$, $U'(x) \in R^N$ with ith component $U_i'(x_i)$. And the link price generation function is given by

$$p = h(y) \tag{14}$$

where $h(y) \in R^L$, with lth component is $h_l(y_l)$ which may be considered as a penalty function enforcing the link capacity constraints, $y_l \leq c_l(t)$. The function $h(y)$ is monotonically nondecreasing and nonnegative which is defined in [1] as $h_l(y) = \dfrac{(y_l - c_l + \varepsilon)^+}{\varepsilon^2}$. Here, considering the dynamic capacity case, introducing a buffering factor δ and dynamic adjusted factor $\gamma(t)$, link price generated function is defined as follows

$$h_l(y_l) = \begin{cases} 0, & \text{当}\, y_l \leq (1-\delta)c_l(t) \\ \gamma(t)(y_l - (1-\delta)c_l(t))^2, & \text{当}\, (1-\delta)c_l(t) < y_l \end{cases} \tag{15}$$

where

$$\delta = \begin{cases} \delta_1, & \dot{c}_l(t) \geq 0 \\ \delta_2, & \dot{c}_l(t) < 0 \end{cases} ,0 < \delta_1 < \delta_2 < 1 . \tag{16}$$

The main motivation to define such a link price generation function above is to reduce the risk of link congestion under consideration of dynamic link capacity case. The buffering factor and dynamic adjusted factor become smaller while available link capacity becomes increasing which aims to widen the buffer size.

But in this case under consideration, from (11) and (12), we can find that equilibrium is shifted and timely variant because link capacity $c(t)$ is not constant now. Therefore, equilibrium manifold can be obtained as

$$q^* = U'(x^*(t)) \quad \text{and } p^* = h(y^*(t)) \tag{17}$$

It is remarkable to note that much attention should be paid to link capacity's changing property such as rate changing velocity, changing shape and so on. The detailed analysis of the influence of dynamic link capacity upon equilibrium manifold and control performance will be done in next research step. In this note, network data flow control shown at Fig. 1 considering the differences between real-time data flow and equilibriums as follows

Proposition 3.1. Assume that: (1) link capacity $c_l(t)$ is dynamic changing with time; (2) If $U : R^N \to R^N$ satisfies $U''(x) < -\delta I_N$, $\delta > 0$, and I_N is $N \times N$ unit matrix. (3). Routing matrix R is of full row rank, then optimality condition given by (11) and (12) has a unique equilibrium manifold (17).

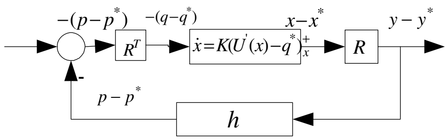

Fig. 1. Diagram of network data flow control in error means

For simplicity some interpretation of the above proposition is given. When the variable t is frozen and denoted as $\bar{t}$, it is apparent to see now the situation is same with the Appendix I in [5]. That is to say, for every $c(\bar{t})$, there are unique equilibrium $p^*(\bar{t})$, $q^*(\bar{t})$, $x^*(\bar{t})$, $y^*(\bar{t})$. So, when link capacity $c_l(t)$ is dynamic changing with time, $p^*(\bar{t})$, $q^*(\bar{t})$, $x^*(\bar{t})$, $y^*(\bar{t})$ forms a dynamic equilibrium manifold each.

Here we focus on the equilibrium manifold over the dynamic link capacity case. And the next task is to realize stability analysis of network flow control for the primal optimal control system and the dual optimal control system.

4 Stability Analysis

Firstly, according to the passivity-based flow control structure shown in Fig. 1, we observe the system with source rate controller and link update law as follows

$$\begin{cases} \dot{x} = K(U'(x) - q)_x^+ \\ p = h(y) \\ y = (\sum\limits_{i \in s(l)} x_i(t))_l = Rx \\ q = (\sum\limits_{l \in l(i)} p_l)_i = R^T p \end{cases} \tag{18}$$

and we have the following results.

Theorem 4.1. Considering the closed-loop system (18) shown in Fig. 1, with the assumption that $U''(x) < -\delta I_N$, for some $\delta > 0$, and the link penalty function $h(y)$ satisfies

$$0 \le h'(y) \le \eta, \text{ for all } y \ge 0 \text{ and all links} \tag{19}$$

where η is a positive constant. Then the two following inequalities hold

$$\begin{aligned} \left\|x - x^*\right\|_{L_p} &\le \sqrt{k_{\max}}(\delta k_{\min} p)^{-1/p} \sqrt{(x(0) - x^*(0)^T K(x(0) - x^*(0)} \\ &\quad + \sqrt{2}\sqrt{k_{\max}}(\delta k_{\min} q)^{-1/q} \left\| 1/\sqrt{2} k_{\max}/k_{\min} \left\| \dot{x}^*(t) \right\| \right\|_{L_p} \end{aligned} \tag{20}$$

$$\left\|p-p^*\right\|_{L_p} \le \eta\|R\|\left\|x-x^*\right\|_{L_p}. \tag{21}$$

That is, if $\left\|\dot{x}^*(t)\right\|$ is L_p, then the system (18) is L_p stable.

Proof: We take the Lyapunov function as follows

$$V_1(x-x^*(t)) = \frac{1}{2}\sum_{i=1,\cdots,N}\frac{(x_i-x_i^*(t))^2}{k_i}. \tag{22}$$

Its derivative along the solution is

$$\begin{aligned}\dot{V}_1 &= \sum_{i=1,\cdots,N}(x_i-x_i^*(t))(\dot{x}_i-\dot{x}_i^*(t))/k_i \\ &= \sum_{i=1,\cdots,N}(x_i-x_i^*(t))(U_i^{'}(x_i)-q_i)_x^+ -(x_i-x_i^*(t))/k_i\dot{x}_i^*(t)\end{aligned} \tag{23}$$

According to the definition of positive projection, it yields

$$(x_i-x_i^*(t))(U_i^{'}(x_i)-q_i)_x^+ \le (x_i-x_i^*(t))(U_i^{'}(x_i)-q_i) \tag{24}$$

With the above inequality, (23) can turn into

$$\begin{aligned}\dot{V}_1 &\le \sum_{i=1,\cdots,N}(x_i-x_i^*(t))(U_i^{'}(x_i)-q_i)-(x_i-x_i^*(t))/k_i\dot{x}_i^*(t) \\ &= \sum_{i=1,\cdots,N}(x_i-x_i^*(t))(U_i^{'}(x_i)-q_i^*+q_i^*-q_i)-(x_i-x_i^*(t))/k_i\dot{x}_i^*(t) \\ &= \sum_{i=1,\cdots,N}(x_i-x_i^*(t))(U_i^{'}(x_i)-U_i^{'}(x_i^*(t))+q_i^*-q_i)-(x_i-x_i^*(t))/k_i\dot{x}_i^*(t) \\ &= (x-x^*(t))^T(U^{'}(x)-U^{'}(x^*))-(x-x^*(t))^T(R^T(p)-R^T(p^*)) \\ &\quad -(x-x^*(t))^TK^{-1}\dot{x}^*(t)\end{aligned}$$

Considering the utilization function is strictly concave, then

$$\begin{aligned}\dot{V}_1 &\le -\delta\|\Delta x\|^2-\Delta x^Tk\dot{x}^*(t)-(y-y^*(t))^T(p-p^*) \\ &= -\delta\|\Delta x\|^2-\Delta x^Tk\dot{x}^*(t)-(x-x^*(t))^TR^T(h(Rx)-h(Rx^*))\end{aligned} \tag{25}$$

And with the property of the function vector satisfies (19), further we have

$$\begin{aligned}\dot{V}_1 &\le -\delta\|\Delta x\|^2+k\|\Delta x\|\left\|\dot{x}^*(t)\right\|-(x-x^*(t)^TR^Th^{'}(\xi)R(x-x^*) \\ &\le -\delta\|\Delta x\|^2+k\|\Delta x\|\left\|\dot{x}^*(t)\right\| \\ &\le -2\delta k_{\min}V_1+\sqrt{2}k_{\max}/k_{\min}\left\|\dot{x}^*(t)\right\|\sqrt{V_1}\end{aligned} \tag{26}$$

where $k = k_{\max}$, $k_{\min} = \min\{k_i, i = 1,\cdots,N\}$, $k_{\max} = \min\{k_i, i = 1,\cdots,N\}$, $\Delta x = x - x^*$ and $\xi \in [Rx, Rx^*]$ which follows from the mean value theorem. We take $W = \sqrt{V_1}$ and obtain

$$D^+ W = -\delta k_{\min} W + 1/\sqrt{2} k_{\max} / k_{\min} \left\| \dot{x}^*(t) \right\| \tag{27}$$

According Lemma 2.1, we get

$$\left\| W \right\|_{L_p} \le (\delta k_{\min} p)^{-1/p} \left\| W(0) \right\| + (\delta k_{\min} q)^{-1/q} \left\| 1/\sqrt{2} k_{\max} / k_{\min} \left\| \dot{x}^*(t) \right\| \right\|_{L_p}$$

and

$$\left\| W(t) \right\| \le e^{-\delta k_{\min} t} \left\| W(0) \right\| + (\delta k_{\min})^{-1} \left\| 1/\sqrt{2} k_{\max} / k_{\min} \left\| \dot{x}^*(t) \right\| \right\|_{L_\infty} .$$

Therefore, following from the above, it is easy to obtain (20) and (21). Furthermore, the system is L_p stable if $\left\| \dot{x}^*(t) \right\|$ is L_p .

Remark 4.1. From (25), it is not difficulty to find that the forward system in Fig.1 is strictly passive when the following

$$\left\| \Delta x \right\| > \frac{k}{\delta} \left\| \dot{x}^*(t) \right\| \tag{28}$$

holds. Further, it shows that for the case of dynamic link capacity the forward system strictly passivity partly depends not only on the approximate difference of information source rate $x(t)$ to the equilibrium dynamic $x^*(t)$ but also on its changing degree. It is not always true that the faster the source flow rate approaches, the better performance of the system.

Remark 4.2. According to theorem 4.1, we know that that the system under consideration is L_p stable depends upon the property of $\left\| \dot{x}^*(t) \right\|$, which in fact upon the property of dynamic link capacity $c(t)$.

Next, we consider the dual problem shown as Fig. 2.

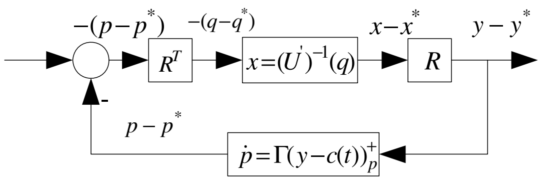

Fig. 2. Diagram of the dual problem in error means

and we define it as follows

$$\begin{cases} x = (U^{'})^{-1}(q) \\ \dot{p} = \Gamma(y - c(t))_p^+ \\ y = (\sum_{i \in s(l)} x_i(t))_l = Rx \\ q = (\sum_{l \in l(i)} p_l)_i = R^T p \end{cases} \tag{29}$$

where $\Gamma = diag\{\lambda_l > 0, l = 1, \cdots, L\}$ □ and denote $\lambda_{\max} = \max\{\lambda_l, l = 1, \cdots, L\}$ and $\lambda_{\min} = \min\{\lambda_l, l = 1, \cdots, L\}$.

Theorem 4.2. For the system (29) shown as Fig.2, assume that

$$-\delta_1 I_N \le U^{''}(x) \le -\delta_2 I_N \tag{30}$$

and for certain positive constant c_2, the following condition

$$\|\dot{p}(t)\| \le c_2 \tag{31}$$

where c_2 is a positive constant, then the system (29) is L_p stable. And furthermore, there exist

$$\|p-p^*\|_{L_p} \le \sqrt{\lambda_{\max}}(\alpha p)^{-1/p}\sqrt{(p(0)-x^*(0)^T\Gamma^{-1}(x(0)-x^*(0)} + \sqrt{2}\sqrt{\lambda_{\max}}(\alpha q)^{-1/q}\|\beta\|_{L_p} \tag{32}$$

$$\|x - x^*(t)\| \le 1/\eta_2 \|R\| \|p - p^*(t)\|_{L_p} \tag{33}$$

where

$$\alpha = \delta_1^{-1}\lambda_{\min}\sigma_{\min}^2(R) \tag{34}$$

$$\beta = 1/\sqrt{2}\lambda_{\max}/\lambda_{\min}\|\dot{p}^*(t)\| \tag{35}$$

Proof: First, define the Lyapunov function as

$$V_2(p - p^*) = \frac{1}{2}\sum_{l=1,\cdots,L}\frac{(p_l - p_l^*(t))^2}{\lambda_l} \tag{36}$$

Along with the (29), the difference of the above can be obtained

$$\begin{aligned} \dot{V}_2 &= \sum_{l=1,\cdots,L}(p_l - p_l^*(t))(\dot{p}_l - \dot{p}_l^*(t))/\lambda_l \\ &= (p - p^*)^T(y - c)_p^+ - (p - p^*)^T\Gamma^{-1}\dot{p}^* \end{aligned}$$

$$\le (p-p^*)^T (y-y^*) - (p-p^*)^T \Gamma^{-1} \dot{p}^* \tag{37}$$

$$= (p-p^*)^T R((U^{'})^{-1}(R^T p) - (U^{'})^{-1}(R^T p^*)) - (p-p^*)\Gamma^{-1}\dot{p}^*$$

$$= (p-p^*)^T R(U^{''})^{-1}(\xi) R^T (p-p^*) - (p-p^*)\Gamma^{-1}\dot{p}^*$$

The above follows from the mean value theorem, where $\xi \in [p, p^*]$ and with the assumption (30), the above turns into

$$\dot{V}_2 \le -\delta_1^{-1} \left\| R^T (p - p^*(t)) \right\|^2 + 1/\lambda_{\min} \left\| p - p^* \right\| \left\| \dot{p}^* \right\|$$

$$\le -2\delta_1^{-1} \lambda_{\min} \sigma_{\min}^2 (R) V_2 + \sqrt{2} \lambda_{\max} / \lambda_{\min} \left\| \dot{p}^* \right\| \sqrt{V_2} \tag{38}$$

Further using $W = \sqrt{V_2}$, we have

$$D^+ W = -\delta_1^{-1} \lambda_{\min} \sigma_{\min}^2 (R) W + 1/\sqrt{2} \lambda_{\max} / \lambda_{\min} \left\| \dot{p}^* (t) \right\|. \tag{39}$$

From Lemma 2.1, it is easy to get (32), and because of

$$x = (U^{'})^{-1}(q) = (U^{'})^{-1}(Rp)$$

and mean value theorem, the (33) is not difficult to be obtained .

Remark 4.3. From (37), only when the following

$$\dot{p}^* = \Lambda (p - p^*), \text{ for } \Lambda = diag\{\tau_l > 0, l = 1, \cdots, L\}, \tag{40}$$

then return subsystem is strictly passive.

Remark 4.4. Through the deep analysis above, the condition stability of the system under consideration is comparatively tough. For dynamic link capacity case, the property of $\left\| \dot{p}^* \right\|$ which is determined by dynamic link capacity, plays very important role of the network data flow control system.

5 Simulations

First, we assume simulation test is made based on the topological graph of network data flows shown in Fig.3,

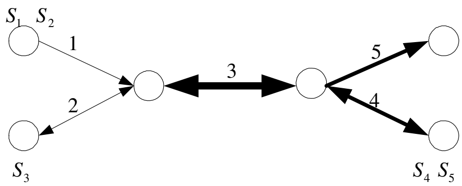

Fig. 3. Topological graph of network data flows

and the routing matrix R is given as follows

$$R = \begin{bmatrix} 1 & 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 1 & 0 \\ 1 & 1 & 1 & 1 & 0 \\ 1 & 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 0 & 1 \end{bmatrix} \tag{41}$$

The buffering factor $\vartheta_1 = 1/8, \vartheta_2 = 1/6$, and dynamic adjusted factor $\gamma(t)$ is chosen as follows

$$\gamma(t) = \begin{cases} 3, & \text{当}\ \dot{c}(t) < 0 \\ 2, & \text{当}\ \dot{c}(t) = 0 \\ 1, & \text{当}\ \dot{c}(t) > 0 \end{cases} \tag{42}$$

Here, two cases are considered in the simulation tests. Firstly, network link capacities available to be occupied for feedback based are constant, with 10, 8, 24, 15 and 16 Mb/sec from link 1 to link 5 respectively. The flow rate of each data flow is shown in Fig.4. Secondly, considering the dynamic link capacity case, that is, the dynamic capacity of link 3 and link 4 given by (43), the simulation test is shown is Fig.5.

$$c_3(t) = \begin{cases} 20\,, & 0 \le t \le 10 \\ 20 + 3t, & 10 \le t \le 15 \\ 15\,, & 15 \le t \end{cases} \qquad c_4(t) = \begin{cases} 30 - t, & 0 \le t \le 15 \\ 20\,, & 15 \le t \end{cases} \tag{43}$$

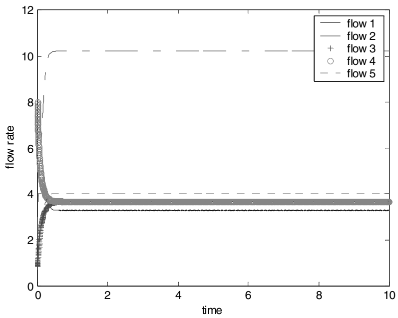

Fig. 4. Flow rate for constant capacity case

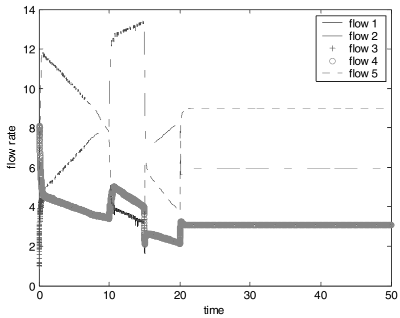

Fig. 5. Flow rate for dynamic capacity case

6 Conclusion

In this paper L_p stability analysis of network data flow control system with dynamic link capacity case from the Lyapunov stability point of view, is first done for both its prime flow control system and its dual control system. Moreover, the corresponding comparison with passive or strictly passive property of flow control system is made. The conclusion is that stability performance of network data flow control system under consideration depends on property of dynamic link capacities. The simulations illustrate the stability analysis in this paper is good and enlightening. Therefore, research about transient and statistical property of dynamic link capacity needs to be made in the fields of control and computer areas.

Acknowledgments

This work is partly supported by the National Natural Science Foundation of China under Grant No. 60334020 and No. 60440420130.

References

1. Kelly, F., Maulloo, A., Tan, D.: Rate Control In Communication Networks: Shadow Prices, Proportional Fairness And Stability. J. Oper. Res. Soc. 49 (1998) 237-252
2. Low, S.H., Lapsley, D.E.: Optimization Flow Control—I: Basic Algorithm And Convergence. IEEE/ACM Trans. Networking. 7 (1999) 861-874
3. Low, S.H., Paganini, F., Doyle J.C.: Internet Congestion Control. IEEE Control System Magazine. 22 (2002) 28-43
4. Johari, R., Tan, D.K.H.: End-to-End Congestion Control for the Internet: Delays and Stability. IEEE/ACM Trans. Networking. 9 (2001) 818-832
5. Wen, J.T., Arcak, M.: A Unity Passivity Framework for Network Flow Control. IEEE Trans. Automatic Control. 49 (2004) 162-174
6. Paganini, F.: A Global Stability Result In Network Flow Control. System Control Letters. 46 (2002) 165-172
7. Ioannou, P., Tao, G.: Frequency Domain Conditions for Strictly Positive Real Functions. IEEE Trans. Automatic Control. 32 (1987) 53-54
8. Wen, J.T.: Time Domain and Frequency Domain Conditions for Strict Positive Realness. IEEE Trans. Automatic Control. 33 (1988) 988-992
9. Paganini, F., Doyle, J.C., Low, S.H.: Scalable Laws for Stable Network Congestion Control. In Proc. of Conference on Decision Control, Orlando, FL. (2001) 185-190
10. Arjan van der Schaft: L_2-Gain and Passivity Techniques in Nonlinear Control. Springer-Verlag London Limited (2000)
11. Khalil, H.: Nonlinear Systems. 2nd ed. England Cliffs, NJ: Prentice Hall (1996)