Dual-tree complex wavelet hidden Markov tree model for image denoising

F.-X. Yan, S.-L. Peng and L.-Z. Cheng

A new non-training complex wavelet hidden Markov tree (HMT) model, which is based on the dual-tree complex wavelet transform and a fast parameter estimation technique, is proposed for image denoising. This new model can mitigate the two problems (high computational cost and shift-variance) of the conventional wavelet HMT model simultaneously. Experiments show that the denoising approach with this new model achieves better performance than other related HMT-based image denoising algorithms.

Introduction: The compact representation of the discrete wavelet transform (DWT) has led to many successful signal and image processing algorithms. By capturing the dependencies between the wavelet coefficients, we can improve the performance of wavelet-based algorithms significantly. Among the many different approaches to modelling the dependencies, the wavelet-domain hidden Markov model is almost the most appropriate [1]. This model can effectively characterise the joint statistics of wavelet coefficients and has been applied to image denoising, segmentation and Bayesian image analysis, etc. But the currently used wavelet-domain HMT model has two disadvantages that undermine its usage in many applications. First, the training of this model needs an iterative expectation-maximisation (EM) algorithm, which results in high computation cost. Next, most image processing algorithms based on this model have a tendency to produce images with mild ringing artefacts around the edges. At the heart of this problem is the fact that the real orthogonal wavelet transform is not shift-invariant. To reduce computation cost of the HMT model, Peng *et al.* [2] recently introduced a fast classification-based parameter estimation technique for the wavelet-domain HMT model, which does not need model parameter training. In [2], wavelet coefficients in each subband were classified into two classes based on spatially adaptive thresholds, and model parameters were estimated by using the local statistics. Redundant wavelet denoising algorithms (including [3], 'cycle spinning' [4], and the undecimated HMT [5]) all tried to solve the problem of shift-variance, but their high transform redundancy incurs a massive storage requirement that makes these undecimated HMT models inappropriate for most applications. Romberg *et al.* [6] also addressed the problem of shift-variance in the wavelet HMT model. They extended the HMT modelling framework to the complex wavelet transform and proposed the complex wavelet HMT model (CHMT). But they still employed the iterative EM algorithm to train a set of CHMT parameters, which is complex and computationally expensive.

In this Letter, we present a new dual-tree complex wavelet HMT model with localised parameters to mitigate the two problems of the conventional wavelet HMT model simultaneously. And we apply this non-training dual-tree complex wavelet HMT model to image denoising to demonstrate its effectiveness.

Complex wavelet HMT model: The DT-CWT is a valuable enhancement to the traditional real DWT, with important additional properties: it is nearly shift invariant and directionally selective in two and higher dimensions. There are six directional subbands capturing features along lines at angles of $\{\pm 15^\circ, \pm 45^\circ, \pm 75^\circ\}$ [7]. By providing explicit information about singularities at a broader range of orientations, the DT-CWT allows us to distinguish between and characterise images that are different in more subtle ways.

In the complex wavelet HMT model, we associate with each complex coefficient $c_i = u_i + jv_i$ a hidden state q_i taking value $m = S$, L with probability mass function (pmf) $p(q_i)$ depending on whether $|c_i|$ is small or large. The persistence of complex wavelet coefficient magnitudes across scale is modelled by linking these hidden states across scale in a Markov tree, which is similar to the wavelet HMT model [1]. Conditioned on $q_i = m$, c_i is Gaussian with mean, $\mu_{i,m}$ and variance $\sigma^2_{i,m}$. Thus, its overall marginal pdf is given by

$$f(c_i) = \sum_{m=S,L} p(q_i = m)\, f(c_i|q_i = m) \tag{1}$$

Consider each complex coefficient c_i as a random vector (u_i, v_i). We approximate the marginal density $f(c_i)$ as a two-state, 2D Gaussian mixture

$$f(c_i|q_i = m) = \frac{1}{\sqrt{2\pi}\sigma_{i,m}} \exp(-u_i^2/2\sigma^2_{i,m}) \exp(-v_i^2/2\sigma^2_{i,m}) \tag{2}$$

The complex wavelet HMT corresponding to (2) with scale-to-scale Markov transitions has an almost identical structure to the real DWT HMT; see [1, 5] for more details. The differences will be the substitution of (2) for (1) and the use of six subband trees instead of three. Using $\rho(i)$ to denote the index of the parent of node i, the parameter $\varepsilon^{m,n}_{i,\rho(i)} = p(q_i = m | q_{\rho(i)} = n)$ gives the probability that a child coefficient c_i has hidden state m when its parent $c_{\rho(i)}$ has state n. The HMT model parameters consist of the Gaussian mixture means and variances, $\mu_{i,m}$, $\sigma^2_{i,m}$ of the complex wavelet coefficient c_i given its state $q_i = m$, the transition probabilities $\varepsilon^{m,n}_{i,\rho(i)}$, and the pmf $P_{S_1}(m)$ for the root node S_1. Generally $\mu_{i,m} = 0$. Group these into the parameter vector $\Theta = \{P_{S_1}(m), \varepsilon^{m,n}_{i,\rho(i)}, \sigma^2_{i,m}\}$. In image denoising, the estimation problem can be expressed in the complex wavelet domain as $c_i = y_i + n_i$, where c_i, y_i and n_i denote the complex wavelet coefficients of the observed data, the signal and the noise, respectively.

If the complex HMT model $\Theta = \{P_{S_1}(m), \varepsilon^{m,n}_{i,\rho(i)}, \gamma^2_{i,m}\}$ for the noise signal is estimated, then the conditional mean estimate of Y_i given c_i is

$$\hat{y}_i = E[y_i|c_i, \Theta] = \sum_{m=S,L} P_{q_i}(q_i = m|c_i, \Theta) \frac{\sigma^2_{i,m}}{\sigma^2_{i,m} + \sigma^2_n} c_i \tag{3}$$

where $\sigma^2_{i,m} = \max\ (0, \gamma^2_{i,m} - \sigma^2_n)$. And then the final signal estimate is computed as the inverse complex wavelet transform of these estimates of the signal complex wavelet coefficients.

Localised parameters estimation for complex wavelet HMT model: We can classify the complex wavelet coefficients into two states, large and small, by introducing the adaptive thresholds T computed by $T = \sigma^2_n/\sigma_s$, where σ^2_n is the additive noise variance and σ_s is the localised standard deviation of the signal. A robust median estimator is used to estimate σ_n:

$$\hat{\sigma}_n = \text{median}(|y_{+45^\circ}|)/0.6745 \tag{4}$$

where $Y_{+45^\circ}\varepsilon$ complex wavelet coefficients of the subband oriented in $\pm 45^\circ$ at the finest scale. The needed localised standard deviation σ_s is computed by an approximate maximum likelihood:

$$\sigma^2_s[i,j] = \max\left(0, \frac{1}{\langle N[i,j]\rangle} \sum_{[k,j]\in N[i,j]} |c[k,j]|^2 - \sigma^2_n\right) \tag{5}$$

where $|c[i, j]|$ denotes the magnitudes of the complex wavelet coefficients, $N[i, j]$ denotes the neighbourhood of the location i in scale j, and $\langle N[i,\ j]\rangle$ denotes the number of coefficients included in $N\ [i,\ j]$ We use a binary mask $M\ [i,\ j]$ to denote the state of the complex coefficients, where '1' denotes 'large' state and '0' denotes 'small' state:

$$M[i,j] = \begin{cases} 0 & |c[i,j]| < T[i,j] \\ 1 & |c[i,j]| \geq T[i,j] \end{cases} \tag{6}$$

Because large values of complex coefficients tend to propagate across scales, $M[i, j]$ is modified by its parent's state:

$$M[i,j] = M[i,j]^* M[\rho(i), j+1] \tag{7}$$

When we have classified the complex wavelet coefficients into two states, we can estimate (with no iterative training) the localised parameters $\Theta = \{P_{S_1}(m), \varepsilon^{m,n}_{i,\rho(i)}, \gamma^2_{i,m}\}$ of our new complex HMT model as follows:

1. State probabilities of the root node S_1 in the coarsest scale J:

$$P_{S_1}(m) = \frac{1}{\langle N[i,j]\rangle} \sum_{[k,J]\in N[i,J]} M_m[k,J],\ m = 0, 1 \tag{8}$$

where $M_m\ [k, J]$ denotes the pixels whose state is m in $M\ [i, j]$.

2. The state transition probability

$$\varepsilon^{m,n}_{i,\rho(i)} = \sum_{[k,j]\in N[i,j]} T_m[k,j]^* T_n[\rho(k), j+1] \Big/ \times \sum_{[k,j]\in N[i,j]} T_n[\rho(k), j+1] \tag{9}$$

3. The state variance of the noisy signal:

$$\lambda_{i,m}^2 = \sum_{[k,j]\in N[i,j]} \left|c[k,j]\right|^2 * T_m[k,j] \Big/ \sum_{[k,j]\in N[i,j]} T_m[k,j] \qquad (10)$$

Using (3), the estimated complex wavelet coefficients are obtained, and the final denoised image is computed by the inverse dual-tree complex wavelet transform.

Experimental results: We tested our algorithm on three standard test images, namely, 'Lena', 'Boats', and 'Bridge' to make a comparison with other HMT-based image denoising algorithms [1, 2, 6]. We have applied five decomposition stages of a dual-tree complex wavelet transform for our denoising procedure. The noise variance σ_n is 25.5. Table 1 gives the output PSNRs and computational times of the various algorithms based on the HMT model. Fig. 1 shows the visual metric of different denoising methods. As can be observed, our algorithm not only gives superior performance in terms of PSNR, but also can reduce computational time significantly. Also, we can see that the subjective quality of the proposed approach is better than other related methods in sharp edges and flat area.

Table 1: Comparisons of PSNR in dB and computational time in seconds of different algorithms

Image	Lena		Boats		Bridge	
	PSNR	Time	PSNR	Time	PSNR	Time
Noisy	20.02		20.05		20.03	
[1]	29.34	265.6	27.68	248.6	25.19	274.8
[2]	30.36	19.74	28.32	20.61	25.41	19.76
[6]	30.53	272.6	29.28	254.0	25.79	285.8
Ours	31.13	19.17	29.67	17.25	26.44	18.45

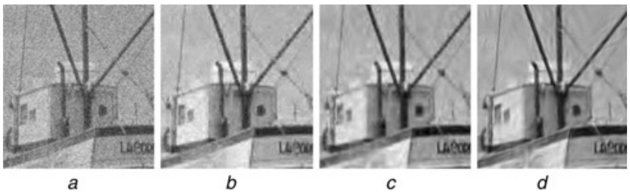

Fig. 1 *Visual metric of different denoising methods*

a Part of noisy 'Boats' image, PSNR = 20.05 dB
b Denoised result using wavelet HMT [1], PSNR = 27.68 dB
c Denoised result using Xiao's algorithm [2], PSNR = 28.32 dB
d Denoised result using proposed method, PSNR = 29.67 dB

Conclusions: A simple and effective dual-tree complex HMT-based image denoising algorithm using fast parameter estimation has been proposed. The competitive performance of the proposed approach is because the estimated parameters are locally adaptive, with non-training, and can characterise the properties of complex wavelet coefficients more accurately; at the same time, the complex wavelet HMT models can capture singularity more accurately than those based on real wavelets.

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F.-X. Yan and L.-Z. Cheng (*School of Science, National University of Defense Technology, Changsha 410073, People's Republic of China*)

E-mail: fxyan79@hotmail.com

S.-L. Peng (*National ASIC Design Engineering Center, Institute of Automation, Chinese Academy of Sciences, Beijing 100080, People's Republic of China*)

F.-X. Yan: Also with the National ASIC Design Engineering Center, Institute of Automation, Chinese Academy of Sciences, Beijing.

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