Fairness and Dynamic Flow Control in Both Unicast and Multicast Architecture Networks

Yuequan Yang, Zhiqiang Cao, Min Tan, and Jianqiang Yi

***Abstract*—With the development of multicast service in the Internet, much attention has been drawn to multicast congestion control and analysis. Multicast traffic poses new challenges to the design of Internet congestion control protocols and system stability analysis. The rate control problem of feedback-based sessions on the coexistence of both unicast and multirate multicast traffic architecture networks is focused upon in this paper. First, a fairness problem is discussed in detail, and a *reasonable consumption* strategy is proposed. In the reasonable consumption strategy, scaling functions are adaptively adjusted based on a relationship between the session rates. Second, contraposing the case that available link capacities are changing with time for these feedback-based unicast and multicast sessions, stability analysis of a closed-loop rate control system under the modified rate mechanism is made based on Lyapunov stable theory. Finally, the simulations illustrate the effectiveness and goodness of the reasonable consumption strategy.**

***Index Terms*—Fairness, flow control, multicast, stability, unicast.**

I. Introduction

WITH the development of multicast service in the Internet, much attention has been drawn toward multicast congestion control and fair utilization of network bandwidth resource. Multicast traffic poses new challenges to the design of Internet congestion control protocols and system stability analysis. The first incentive of promoting multicast service is to maximize the network resource utilization. It is in the interest of the network resource to encourage the use of multicast, because this service demands fewer network resources than that of the corresponding separate unicast sessions. The second incentive is to satisfy the increasing demand of both concurrent transmission and live broadcast service. For example, network layered video service is a popular application. Multicast is, in fact, a service in which packets/streams (hereafter, packets/streams are denoted as flows) from one source can be transmitted simultaneously to a group of destination terminals. At junction nodes, flows are copied and sent to different downstream links in multicast networks. Then, the problem arises as to how to decide the transmitting rates of flows for maximum utilization of network link bandwidth resource and best realization of multicast service. As a result, multicast congestion control and fair link bandwidth allocation become the two most important difficulties.

The multicast congestion control task is to detect possible congestion occurrence and regulate the source flow rate to be adapted to the available bandwidth. Golestani and Sabnani [1] have studied the choice of regulation parameter for multicast congestion control, the fairness implications of this choice, and the scalable implementation of rate-based or window-based algorithms using a receiver-driven approach. Zhang and Shin [2] have made an in-depth analysis of the feedback congestion marking and delay signaling problem by developing two models, one of which is the *Markov-chain model* defined by the link marking state on each path in the multicast tree, and the other is the *Markov-chain dependency-degree model* to evaluate all possible Markov-chain dependency degrees without any prior knowledge. Contraposing the above two models, the general probability distribution of each path becoming a bottleneck is derived. There are two kinds of multicast protocols: *unirate multicast* protocol that permits all subflow rates in a multicast session to be identical to the source transmission rate on every link; *multirate multicast* protocol that permits the subflow rates in a multicast session to vary depending on the bandwidth available to downstream branches. A comparison between them shows that the rate of unirate multicast is limited by the most congested link among its subflow paths, while the rate of multirate multicast is decided by the respective congested links on their subroutes. In [3], the optimization-based unified congestion control scheme is provided for networks, where unicast and multirate multicast sessions are shared together with the concept of *virtual sessions*. For unicast sessions, the sending rate is regulated with congestion marking probability. For multirate multicast sessions, where different destination receivers might have different rates, each receiving rate for different receivers is adjusted according to the congestion marking function on its own route. In [4], a unirate multicast congestion control approach is proposed with the help of a receiver-oriented session utility, and unfairness to unicast is considered mainly by tuning the parameters of utilization functions. Deb and Srikant [5] have proposed a new marking and unmark mechanism for multirate multicast sessions, and the stability analysis and different layer simulation tests have been done. Kelly *et al.* [6] have proposed so-called proportional fairness criteria, and have made in-depth stability analysis of the network optimization problem in single and dual forms.

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Moreover, fairness analysis and implementation are very complex problems for competing users of the unicast and multicast sessions in the Internet. One of the common arguments for fairness is that the more network resources are used, the cost paid by the users in unicast and receivers in multicast topology increased, which apparently results in the fact that the farther a session passes, the less will be the flow rate allocated. It is sometimes actually unfair for receivers far away from the flow transmitting source and users with long route unicast sessions. Based on the mark and unmark idea in [5], a reasonable consumption scheme is proposed. In addition, this is considered in the case when the link capacities are not constant, but changing with time, because of the fact that all link capacities are not occupied by sessions with a feedback mechanism, but some occupied by sessions with an open-loop mechanism. On the contrary, its influence upon system stability is considered here. The system stability, to some extent, depends on the characteristic of the dynamics of link changing capacities. Based on the contribution of previous literature in this field, the stability analysis for the unirate and multirate multicast sessions systems is done by using the Lyapunov stability theory.

This paper is organized as follows. The problem formulation is given in Section II. A fairness scheme called reasonable consumption is proposed in Section III. In Section IV, stability analysis is carried out for the complex dynamic system consisting of unirate and multirate multicast sessions based on a feedback-transmission mechanism. In Section V, the simulation tests are performed, while the conclusions are drawn in Section VI.

II. Problem Formulation

In this paper, we assume the network architecture as follows. It has a set L of links, and $c_l(t)$ is the dynamic capacity of link l. Let S be the set of sessions and R_S be the set of receivers corresponding to any session $s \in S$. Apparently, R_S is a singleton set for a unicast session. S_l denotes the set of all sessions passing through the link l in L. For a receiver r of multicast session from the source s, this session called a *virtual session* [5], written as (s,r), passes the set of all links denoted as $L_{(s,r)}$. Thus, unicast may be a special form of multicast. The rate allocation problem can be converted into optimization problem [5], [6] as

$$\max \sum_{s\in S}\sum_{r\in R_s} \Delta_{sr} U_{sr}(x_{sr})$$

$$\text{s.t.} \quad \sum_{s\in S_l} \max_{(s,r)\in V_{sl}} x_{sr} \le c_l, \quad \forall l \in L \tag{1}$$

$$x_{sr} \ge 0, \quad \forall s \in S, \quad r \in R_s \tag{2}$$

where $U_{sr}(x)$ is the utility function of the virtual session (s,r),V_{sl} is the set of all virtual sessions of the multicast session s, x_{sr} is the flow rate of session (s,r), and Δ_{sr} denotes the scaling function of the subscriber of session (s,r). Some assumptions are (similar to those made in [5]) as follows.

Assumption 2.1:

1) The utility function $U_{sr}(x_{sr})$ is strictly a concave and differentiable increasing function.
2) Topology of unicast and multicast sessions is invariant.
3) For the link through which the multicast session passes, the multicast session rate is the maximum of the rates for all downstream receivers of the multicast session.

Following the optimization theory method and using the multiplier p_l, (1) and (2) can be written as

$$\min_{p_l\ge 0}\max_{x_{sr}} \sum_{s\in S}\sum_{r\in R_s} \Delta_{sr} U_{sr}(x_{sr}) - \sum_{l\in L} p_l \left[\sum_{s\in S_l} \max_{(s,j)\in V_{sl}} x_{sj} - c_l\right] \tag{3}$$

where p_l, also representing the price [6] of link l, has the property of a monotonically nondecreasing function with respect to the total flow rate in the link l, which is different from the packet congestion marking probability. Applying optimal theory and technique, the above expresssion (3) becomes

$$\min_{p_l\ge 0}\max_{x_{sr}} \sum_{s\in S}\sum_{r\in R_s} \left[\Delta_{sr} U_{sr}(x_{sr}) - x_{sr} \sum_{l\in L_{sr}\, \max_{(s,j)\in V_{sl}} x_{sj}=x_{sr}} p_l\right] + c_l \sum_{l\in L} p_l. \tag{4}$$

Because of the nondifferential property of $\sum_{s\in S_l} \max_{(s,j)\in V_{sl}} x_{sj}$ in (3), Deb and Srikant [5] chose the approximation function

$$\sum_{s\in S_l}\left(\sum_{(s,j)\in V_{sl}} x_{\mathrm{sj}}^n\right)^{1/n}$$

with a large enough integer n to substitute for it. Therefore, (3) becomes

$$\min_{p_l\ge 0}\max_{x_{sr}} \sum_{s\in S}\sum_{r\in R_s} \left[\Delta_{sr} U_{sr}(x_{sr}) - \left(\sum_{(s,j)\in V_{sl}} x_{sj}^n\right)^{1/n} \sum_{l\in L_{sr}} p_l\right] + c_l \sum_{l\in L} p_l. \tag{5}$$

The price function used here is the same with [6]

$$p_l = h_l(y_l, c_l(t)) = \frac{(y_l - c_l(t) + \varepsilon)^+}{\varepsilon^2} \tag{6}$$

for certain small positive real $\varepsilon > 0$. We have

$$y_l = \sum_{m\in S_l} \max_{(m,j)\in V_{ml}} x_{mj}. \tag{7}$$

According to the Karush–Kuhn–Tucher condition of the optimization theory, we have

$$\Delta_{sr} U'_{sr}(x_{sr}) - x_{sr}^{n-1} \left(\sum_{(s,j)\in V_{sk}} x_{sj}^n\right)^{1/n-1} \sum_{l\in L_{sr}} p_l = 0. \tag{8}$$

Let

$$q_s = \sum_{l\in L_{sr}} p_l \quad G_{sr} = x_{sr}^{n-1}\left(\sum_{(s,j)\in V_{sk}} x_{sj}^n\right)^{1/n-1}.$$

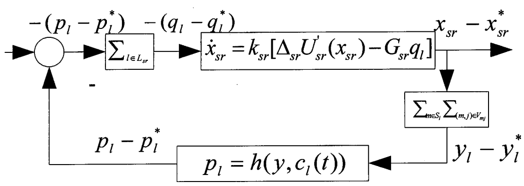

Fig. 1. Diagram of the rate control problem.

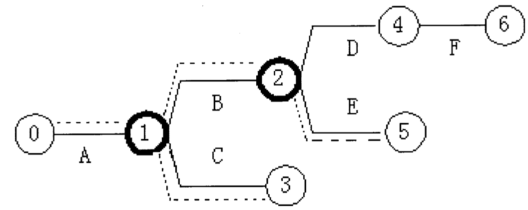

Fig. 2. Topology graph with unicast and multicast sessions. (Real line) Multicast. (Dotted line) Unicast. (Thick circle) Junction node.

Then, at the equilibrium denoted as (x_{sr}^*, p_l^*), it yields

$$\Delta_{sr} U'_{sr}(x_{sr}^*) = G_{sr} q_l^* \tag{9}$$

if the flow rate control law is chosen as follows:

$$\dot{x}_{sr} = k[\Delta_{sr} U'_{sr}(x_{sr}) - G_{sr} q_l]. \tag{10}$$

Thus, from a control theory point of view [6], the task (5) might be described as in Fig. 1.

It is clear that when computing G_{sr}, all other virtual sessions rates needs to be known. Thus, it is true that (10) and the rate control law in [5] and (8) are not decentralized. One of the advantages is the convenience of theoretical analysis of the rate performance.

In this paper, the first objective is to design a modified rate control strategy called a reasonable consumption fairness scheme based on the method in [5]. The second one is to make an in-depth stability analysis of the feedback-based rate control system. And more, we will investigate the multicast flow rate dynamics under the above assumptions that the link capacity available for feedback-based sessions is not constant, but changing with time. And further, its influence upon network system performance is considered. For the realization of the strategy of fairness scheme proposed in Section III, we make some mild assumptions.

Assumption 2.2:

1) For each receiver/subscriber, its route length and number of congestion links are known.
2) The link capacity for sessions with flow rate based on feedback mechanism changes with time, denoted as $c_l(t)$, and is differentiable. For convenience, we further give the following.

Definition 2.1. Positive projection $(g(x))_x^+$ with some function $g(x)$ is defined as

$$(g(x))_x^+ = \begin{cases} g(x), & \text{if } x > 0, or\ x = 0 \text{ and } g(x) \geq 0 \\ 0, & \text{if } x = 0 \text{ and } g(x) < 0. \end{cases} \tag{11}$$

III. Fairness and Reasonable Consumption

Here, we suppose that weighted or scaling function is

$$\Delta_{sr} = f_{sr}(n_{sr}, n_{sr}^c, o_{sr}) \tag{12}$$

where n_{sr} and n_{sr}^c are the length of session (s, r) and the number of bottleneck links on path (s, r) respectively, and o_{sr} is assumed as a parameter representing session priority, time delay, or anything else for some users and receivers.

In Fig. 2, though the unicast (0, 5) passes through more links than (0, 2) does, the rate consumption of the former might be greater than the latter, if the former is willing to pay the expensive price mainly by choosing the proper scaling function and proper utility function as discussed in [4] and [5]. The case is also true for the multirate multicast session. So, fairness has a relative relationship, not an absolute one. To a certain extent, it depends on the capability of consumption of each user and receiver. Therefore, it is not absolutely reasonable to ensure that the rate of unicast from node 0 to node 5 be smaller than the rate from node 0 to node 2. In the networks, every user or receiver has the right to share the same quality of service, which does not completely depend on the distance from source position to destination nodes. For example, let

$$f_{sr}(n_{sr}, n_{sr}^c, o_{sr}) = \frac{k n_{sr}}{n_{sr}^c} \tag{13}$$

which shows that the scaling function value is directly proportional to n_{sr} and inversely proportional to n_{sr}^c. Of course, there may be other forms of definition.

As discussed in [5], only when the virtual session rate equals the highest rate in a multicast session, its price does not equal zero. Therefore, in Fig. 2, if the rate of the virtual session (0, 3) is slower than the required or subscribed rate of the virtual session (0, 6), it is apparent that $x_{0,3}$ will be increasing! What is the result in the end? We know that the virtual session (0, 3) does not need to pay any cost for the session service, and that the bill for its consumption is paid by *the biggest subscriber* at the joint nodes. We call this phenomenon *the biggest treating all*. If this trend goes on, the rate of the virtual session (0, 3) will become the highest in some time. If this happens, the virtual session (0, 6) need not pay the cost, which it did as before. So, after that, the rate of the virtual session (0, 6) will further increase than before. In the end, the worst result is the unfairness for the unicast session or other multicast sessions. The main reason is that all costs of the unicast session must be paid by the unicast session itself. While the link price becomes higher, the unicast must pay more cost. Contraposing this problem, a modified rate control scheme based on the work in [5] is proposed in this paper, which is called reasonable consumption.

In brief, reasonable consumption can be interpreted as no-free and no-discriminated consumption for all session subscribers in the unicast and multirate multicast architecture networks. For a virtual session subscriber, the cost at the sharing links paid by the highest subscriber should be considered, though its flow rate at this moment is not the highest in the shared links. The partaking part should be proportional to the service it shares in the network or multicast service. The reason is that it must pay more cost when attempting to exceed the current highest rate in a multicast session. But another problem appears. If potential rate of one virtual session is impossibly larger than the current multicast rate, e.g., supposing that the rate of multicast session at the links A and B in Fig. 2 now is 4 mps, and the largest feasible capacity in the link E is 2 mps. And consider the rate of the virtual session $(0, 5)$ is never the highest in this multicast session. In this case, the rate control of the virtual session $(0, 5)$ should pay little attention upon this link A and the link B as that in [5]. Unfortunately, in this situation, when there is a unicast session $(0, 5)$ and the virtual session $(0, 5)$ running simultaneously and if they have the same utility function, the occupied flow rate for this unicast is often possibly slower than that of the virtual session, which then results in unfairness to the unicast session. One solution to it is to choose different utility functions and scaling functions, which exhibits the subscriber's willingness to pay. Next, we give a modified flow rate control mechanism as follows.

Algorithm 3.1. In this paper, we consider network architecture with the coexistence of both unicast and multicast sessions flows. For a multicast session, all virtual sessions with different scaling functions are as in (13). For every subscriber, the rate regulation law is regulated as follows.

Case 1: If a virtual session shares only the rates of all the other virtual session subscribers for the same multicast service on its transmission path, and excludes other unicast or multicast sessions, then

$$\dot{x}_{sr} = \dot{\bar{x}}$$

where $\bar{x}$ is assumed the highest rate of corresponding link of a multicast session.

Case 2: When a virtual session shares all current highest rates of other virtual sessions on its own route, its link price should not equal zero, i.e., the current price must be paid. Thus, the rate for it is

$$\dot{x}_{sr} = k_{sr}[\Delta_{sr} U'_{sr}(x_{sr}) - \beta q_s].$$

Case 3: For a virtual session, if it excludes other sessions (unicast session or other multicast sessions) from a certain junction node m to its receiver terminal and its rate is not multicast rate, then only the price of this part of the route may be considered. So the rate control for it is

$$\dot{x}_{sr} = k_{sr}[\Delta_{sr} U'_{sr}(x_{sr}) - q_m]$$

where $q_m = \sum_{l \in (m,r)} p_l$.

Case 4: The cost of a virtual session, which shares all routes with other unicast sessions, should include prices on all its routes, at least include the part proportional to its sharing service. Then, the rate control becomes

$$\dot{x}_{sr} = k_{sr}[\Delta_{sr} U'_{sr}(x_{sr}) - \beta q_s]$$

where q_s may be chosen as $\sum_{l \in L_{sr}} x_{sr}/\bar{x} p_l$.

Case 5: If the multicast rate changes, the corresponding scaling functions might alter with it to follow the new flow rate relationship.

According to Algorithm 3.1, the values of the parameters of the scaling function are adjusted to the virtual session flow rate. So, the rate regulation formula is hard to express in a normalized form. Considering the nonnegative of flow rate, we let

$$\dot{x}_{sr} = k_{sr}[\Delta_{sr} U'_{sr}(x_{sr}) - \beta q_\alpha]^+_x \tag{14}$$

where $\alpha = i$ represents the case $i (i = 1, 2, 3, 4, 5)$, β is a tuning factor, $q_\alpha = \sum_{l \in L_\alpha} p_l$ where L_α denotes the set of links corresponding to the case α.

IV. Stability Analysis

First, we introduce a lemma as follows.

Lemma 4.1. [7] Suppose that $W : [0, \infty) \to R$ satisfies

$$D^+ W(t) \leq -\alpha W(t) + \beta(t)$$

where D^+ denotes the upper Dini derivative, α is a positive constant, and $\beta \in L_p, p \in (1, \infty)$; then

$$\|W\|_{L_p} \leq (\alpha p)^{-1/p} \|W(0)\| + (\alpha q)^{1/q} \|\beta\|_{L_p}$$

where p and q satisfy $1/p + 1/q = 1$. When $p = \infty$, the following estimate holds:

$$\|W\| \leq e^{-\alpha t} \|W(0)\| + \alpha^{-1} \|\beta\|_{L_\infty}.$$

According to game theory, Nash equilibrium exists applying the regulations discussed in the above section with the given mild assumptions. For simplicity, the equilibrium state of the rate control system under consideration is supposed to be

$$x_s^* = (x_{s1}^*, x_{s2}^*, \ldots, x_N^*)^T \quad x^* = (x_1^{*T}, x_2^{*T}, \ldots, x_{|S|}^{*T})^T \tag{15}$$

where N is assumed as the virtual session number for multicast service from the source S, and S denotes the number of the multicast service simultaneously in networks. To be consistent with the above sections, we denote x as (x_{sr}) in the component. In contrast to the situation in [5], the scaling functions in this paper are alterated, and the rate control algorithm is modified. When the highest rate changes to another virtual session, scaling functions will change, and consequently, the set of links for which the subscriber has to pay might change too. The function [5]

$$V(x) = -\frac{1}{k} \sum_{s \in S} \sum_{r \in R_s} \Delta_{sr} U_{sr}(x_{sr}) + \frac{\beta}{k} \sum_{l \in L} \int_0^{\sum_{m \in S_l} \max_{(m,j) \in V_{ml}} x_{mj}} p_l(u) du]$$

is not differentiable because of $\sum_{m \in S_l} \max_{(m,j) \in V_{ml}} x_{mj}$. Therefore, we define a new Lyapunov function based on the difference between point x and equilibrium point vector x^* as

$$V_d = \frac{(x - x^*)^T K^{-1} (x - x^*)}{2}. \tag{16}$$

For convenience, let

$$\Delta U' = \text{diag}\{\Delta_{sr} U'_{sr}\} \quad K = \text{diag}\{k_{sr}\}$$

$$H = \text{diag}\left\{\sum_{l \in L_\alpha} \beta h_l(y_l, c_l(t))\right\} \quad U''(x) = \text{diag}\left\{U''_{sr}(x_{sr})\right\}.$$

Theorem 4.1. Considering the closed-loop system shown in Fig. 1, with the assumption that

$$U''(x) < -\delta I_N, \quad \exists \delta > 0$$

and the link penalty/price function $h(y)$ satisfies $0 \leq h'(y) \leq \eta$, for all $y \geq 0$ and $c_l(t)$ of all links, where η is a positive constant, and the rate control law is (14), then the following inequalities hold:

$$\begin{aligned} \|x - x^*\|_{L_p} \leq & \sqrt{k_{\max}}(\delta k_{\min} p)^{-1/p} \\ & \times \sqrt{(x(0) - x^*(0))^T K (x(0) - x^*(0))} \\ & + \sqrt{2}\sqrt{k_{\max}}(\delta k_{\min} q)^{-1/q} \\ & \times \|1/\sqrt{2} k_{\max}/k_{\min} \|\dot{x}^*(t)\|\|_{L_p}. \end{aligned} \tag{17}$$

Thus, if $\|\dot{x}^*(t)\| \leq c$, where c is certain positive constant, the system is L_p stable.

Proof: From (16), and considering that link capacity for all feedback-based sessions changes with time, we denote the equilibrium as $x^*_{sr}(t)$; then

$$V_d(x - x^*(t)) = \frac{1}{2} \sum_{sr} \frac{(x_{sr} - x^*_{sr}(t))^2}{k_{sr}}. \tag{18}$$

Its derivative along (14) is

$$\begin{aligned} \dot{V}_d &= \sum_{sr} (x_{sr} - x^*_{sr}(t))(\dot{x}_{sr} - \dot{x}^*_{sr}(t))/k_{sr} \\ &= \sum_{sr} (x_{sr} - x^*_{sr}(t))(\Delta_{sr} U'_{sr} - \beta q_\alpha)^+_x \\ &\quad - (x_{sr} - x^*_{sr}(t))\dot{x}^*_{sr}(t)/k_{sr}. \end{aligned} \tag{19}$$

According to Definition 2.1 of positive projection, it yields

$$\begin{aligned} (x_{sr} - x^*_{sr}(t))(\Delta_{sr} U'_{sr} - \beta q_\alpha)^+_x \leq (x_{sr} - x^*_{sr}(t)) \\ \times (\Delta_{sr} U'_{sr} - \beta q_\alpha). \end{aligned}$$

With the above inequality, (19) becomes

$$\begin{aligned} \dot{V}_d &= \sum_{sr} (x_{sr} - x^*_{sr}(t))(\Delta_{sr} U'_{sr} - \beta q_\alpha) \\ &\quad - (x_{sr} - x^*_{sr}(t))\dot{x}^*_{sr}(t)/k_{sr} \\ &= \sum_{sr} (x_{sr} - x^*_{sr}(t))(\Delta_{sr} U'_{sr} - \beta q^*_\alpha + \beta q^*_\alpha - \beta q_\alpha) \\ &\quad - (x_{sr} - x^*_{sr}(t))\dot{x}^*_{sr}(t)/k_{sr} \\ &= \sum_{sr} (x_{sr} - x^*_{sr}(t))[(\Delta_{sr} U'_{sr} - \Delta_{sr} U^*_{sr}) \\ &\quad - (\beta \sum_{l \in L_\alpha} [h_l(y_l, c_l(t)) - h_l(y^*_l, c_l(t))]]_{sr} \\ &\quad - (x_{sr} - x^*_{sr}(t))\dot{x}^*_{sr}(t)/k \\ &= (x - x^*)^T (\Delta U' - \Delta U'(x^*)) \\ &\quad - (x - x^*)^T (H - H^*) - (x - x^*)^T K^{-1} \dot{x}^* \end{aligned}$$

where

$$\begin{aligned} \Delta U'(x^*) &= \mathrm{diag}\,\{\Delta_{sr} U'_{sr}(x^*_{sr})\} \\ H^* &= \mathrm{diag}\left\{\sum_{l \in L_\alpha} \beta h_l(y^*_l, c_l(t))\right\}. \end{aligned}$$

With the assumption that the utilization function is strictly concave, there exists

$$\begin{aligned} &\dot{V}_d \leq -\Gamma\delta\|(x - x^*)\|^2 - (x - x^*)^T K^{-1} \dot{x}^* \\ &- (y - y^*(t))^T (p - p^*) = -\Gamma\delta\|x - x^*\|^2 - (x - x^*)^T K^{-1} \dot{x}^* \\ &- (R_{\max} x - R^*_{\max} x^*)^T (h(R_{\max} x) - h(R^*_{\max} x^*)) \end{aligned} \tag{20}$$

where $\Gamma = \min\{\Delta_{sr}\}$, and $R_{\max}$ is called relation matrix, which represents the relationship between subscribers and links, that is, a component of $R_{\max} = (r_{lr})_{L \times (N*|S|)}$ is

$$r_{lr} = \begin{cases} 1, & \text{rate of subscriber r is the highest in l} \\ 1/\bar{x}, & \text{rate of virtual session r is not the highest in l} \\ 0, & \text{others} \end{cases}$$

and $R^*_{\max}$ is the relation matrix at the system equilibrium state. Further, we have

$$\begin{aligned} \dot{V}_d &\leq -\Gamma\delta\|x - x^*\|^2 - (x - x^*)^T K^{-1} \dot{x}^* \\ &\leq -\Gamma\delta\|x - x^*\|^2 + 1/k_{\min}\|x - x^*\|\|\dot{x}^*(t)\| \end{aligned} \tag{21}$$

where $k_{\min} = \min\{k_{sr}\}$, and (21) follows from the fact that for each link l, there is the following inequality:

$$(R^l_{\max} x - R^{l*}_{\max} x^*)(h(R^l_{\max} x) - h(R^{l*}_{\max} x^*) \geq 0 \tag{22}$$

with the assumption of the property of the function vector $h(\cdot)$. Therefore, (21) yields

$$\dot{V}_d \leq -2\Gamma\delta k_{\min} V_d + \sqrt{2} k_{\max}/k_{\min}\|\dot{x}^*(t)\|\sqrt{V_d}$$

where $k_{\max} = \max\{k_i\}$. Let $W = \sqrt{V_d}$, and it yields

$$D^+ W = -\Gamma\delta k_{\min} W + 1/\sqrt{2} k_{\max}/k_{\min}\|\dot{x}^*(t)\|. \tag{23}$$

According to Lemma 4.1, we easily have

$$\begin{aligned} \|W\|_{L_p} \leq & (\Gamma\delta k_{\min} p)^{-1/p}\|W(0)\| \\ & + (\Gamma\delta k_{\min} q)^{-1/q}\|1/\sqrt{2} k_{\max}/k_{\min}\|\dot{x}^*(t)\|\|_{L_p} \end{aligned}$$

and when $p \to \infty$, we have

$$\begin{aligned} \|W(t)\| \leq & e^{-\Gamma\delta k_{\min} t}\|W(0)\| + (\Gamma\delta k_{\min})^{-1} \\ & \times \|1/\sqrt{2} k_{\max}/k_{\min}\|\dot{x}^*(t)\|\|_{L_\infty}. \end{aligned}$$

Therefore, it is easy to obtain (17) from the above. Furthermore, the system is L_p stable when optimal source rate manifold satisfies $\|\dot{x}^*(t)\| \leq c$ for some positive constant c.

Remark 4.1: In (20), $R_{\max}$ is not constant because of the links that will be included to account for the total prices for a subscriber, which are changing with the status of flow-rates relationship between all kinds of sessions.

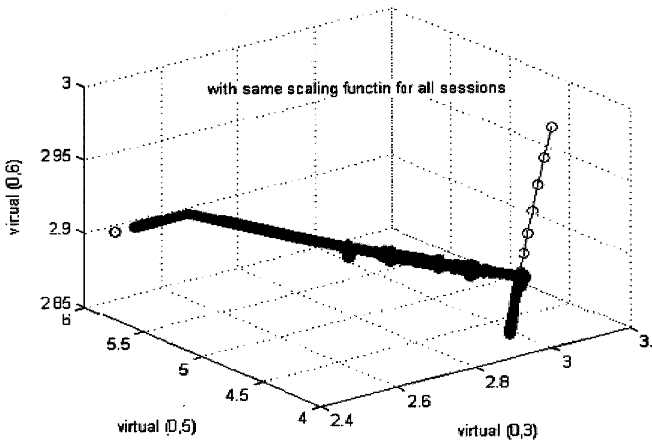

Fig. 3. Rate evolutions of the three virtual sessions.

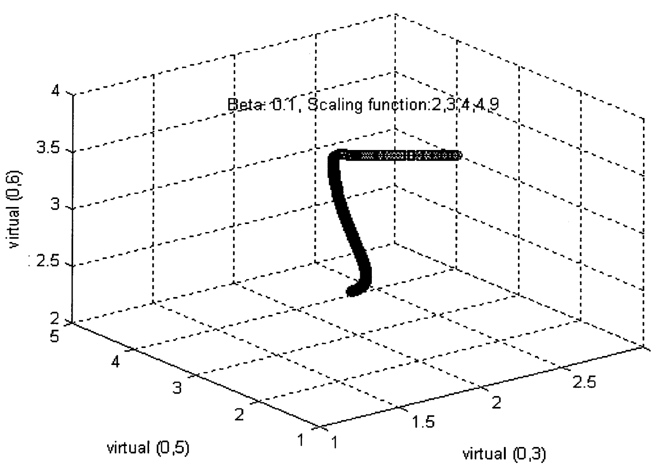

Fig. 4. Rate evolutions with new scaling functions.

Remark 4.2: According to Theorem 4.1, we know that the rate control system under consideration is L_p stable depending upon the property of $\|\dot{x}^*(t)\|$, which, in fact, relies on the property of dynamic link capacity $c_l(t)$ which is available for these feedback-based mechanism sessions. Its influence under such circumstances is partly exhibited in Section V.

V. Simulations

In this section, the combination network services of unicast sessions and multicast sessions together are as shown in Fig. 2, and simulations are made using Matlab. For simplicity, we consider a multicast session which includes $\{(0,3),(0,5),(0,6)\}$ and two unicast sessions which are (0, 3) and (0, 5). First, suppose that the initial rate states are 3, 4, 3, 2, 4 for the virtual session (0, 3), (0, 5), (0, 6) and two unicast sessions (0, 3) and (0, 5), respectively, and static link capacities are assumed as $\{\text{Link A}: 10; \text{Link B}: 15; \text{Link C}: 5; \text{Link D}: 8; \text{Link E}: 6; \text{Link F}: 3\}$. With rate control scheme proposed in [5], we can find that virtual session (0, 5) almost monopolizes all the resources of link E in Fig. 3. Next, the scaling function is chosen as

$$f_{sr}(n_{sr}, n_{sr}^c, o_{sr}) = \begin{cases} n_{sr}, & \text{for virtual session}(s,r) \\ n_{sr}^2, & \text{for unicast session}(s,r) \end{cases}$$

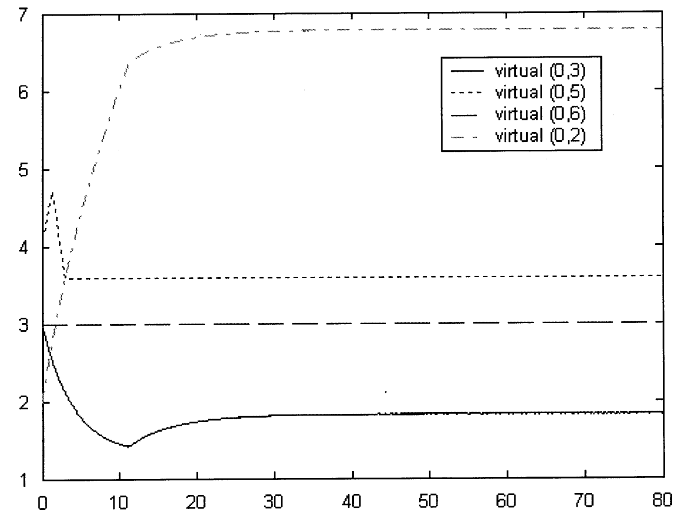

Fig. 5. Rate evolutions with 4th virtual session (0,2).

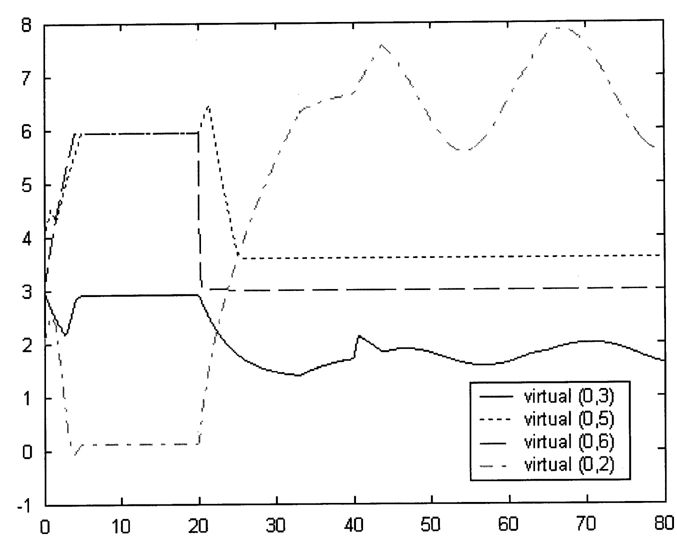

Fig. 6. Rates of virtual sessions with changing capacity of links.

and $\beta = 0.1$. Fig. 4 shows the evolution of flow rates of virtual sessions, while fairness becomes better than before. With the Reasonable Consumption scheme, a good fairness property can be demonstrated in Fig. 5. Moreover, consider that the link capacities for such sessions are changing with time and the link capacities are $\{\text{Link A}: 8; \text{Link B}: 10; \text{Link C}: 5; \text{Link D}: 6; \text{Link E}: 8; \text{Link F}: 6\}$ and $\{\text{Link A}: 10; \text{Link B}: 15; \text{Link C}: 5; \text{Link D}: 8; \text{Link E}: 6; \text{Link F}: 3\}$ at [0 s, 20 s] and [20 s, 40 s], respectively, and after 40 s, the disturbance on the available links capacity is assumed as $1/5\sin((t-60)/4)$. The evolution of session rates is shown in Fig. 6 in this case. Under a dynamic link capacity circumstance, the rates of all the sessions have a very complex property. Note that the labels of all axes in Figs. 3 and 4 and the labels of the vertical axes in Figs. 5 and 6 are flow rates (with unit megabytes per second), and that the labels of the horizontal axes in Figs. 5 and 6 show time in seconds.

VI. Conclusion

In this paper a flow rate control problem on the combination of both unicast and multirate multicast traffic architecture networks is considered for feedback-based mechanism session services. Fairness is deeply discussed, and the strategy called reasonable consumption is proposed. Scaling functions are adaptively adjusted based on a relationship between the session rates. Moreover, contraposing the case that available capacities of links change with time, stability analysis of the rate control system with the modified rate control scheme is made based on Lyapunov stable theory. The simulations show the effectiveness and goodness of the reasonable consumption strategy. Much work such as fairness, time delay, nonstatic topology, etc., needs to be studied in the near future.

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