EMD Sifting Based on Bandwidth

Bo Xuan, Qiwei Xie, and Silong Peng

***Abstract*—Empirical-mode decomposition (EMD) provides a powerful tool for adaptive multiscale analysis of nonstationary signals. Aiming at the intrinsic mode function (IMF) criteria in sifting process and the scale mixing problem in EMD, this paper proposes a bandwidth criterion for IMF. By analyzing the simulated signal, it is confirmed that the IMFs obtained with the bandwidth criterion approximate the real components better and reflect the intrinsic information of the analyzed signal. Furthermore, the criterion based on bandwidth can weaken the scale mixing problem.**

***Index Terms*—Empirical-mode decomposition (EMD), intrinsic mode function (IMF), instantaneous frequency (IF), local narrow-band signal.**

I. Introduction

EMPIRICAL-MODE decomposition (EMD), introduced by N. E. Huang *et al.* [1] in 1998, is a method for decomposing complex, multicomponent signal into several elementary intrinsic-mode functions (IMFs). Unlike Fourier analysis and wavelet analysis, which have predefined bases, EMD only uses the original signal. From this point of view, EMD is a local, fully data-driven and self-adaptive analysis approach [1]–[3]. Moreover, the combination of EMD and the associated Hilbert spectral analysis can offer a powerful method for time–frequency analysis.

Although it has been proved remarkably effective in many applications [1]–[5], EMD has many problems. Besides the mathematical model and problems during the sifting process (SP) (such as boundary effect, overshoots, undershoots, etc.), there are two major problems about IMF. The first is scale mixing: IMF often contains local oscillations with dramatically different frequencies [2]. The second is IMF criteria problem. IMF criteria determine how to select IMF and when to stop a sifting process in EMD. But it is a pity that all of the criteria considered so far [5]–[7] are constraints on the amplitude and are unrelated to frequency and phase information. So IMF obtained based on those criteria would have dramatically different frequencies, and the associated Hilbert spectral analysis would have light meaning.

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This paper proposes the bandwidth criterion for IMF, aiming at getting a better approximation to the real component of the analyzed signal and weakening the scale mixing problem in EMD.

II. EMD Basics

In order to make its instantaneous frequency (IF) have meaningful interpretation, the IMF has to satisfy two conditions [1]: 1) in the whole data set, the number of extrema and the number of zero-crossings must either be equal or differ, at most, by one and 2) at any point, the local average of upper and lower envelope is zero. They are necessary conditions for reasonable IF and instantaneous amplitude.

EMD uses sifting process to extract IMFs from the analyzed signal. We will present EMD in brief, and details are available in [6] and [8].

A. Sifting Process

Given a real valued signal $x(t)$, let $r(t) = x(t), k = 1, i = 0$, the process of EMD can be summarized as follows.

1) Find all local minima and maxima of $r(t)$.
2) Get the upper envelope $e_{\max}(t)$ by interpolating between maxima. Similarly get the lower envelope $e_{\min}(t)$ with minima.
3) Compute the mean envelope as an approximation to the local average $m(t) = (e_{\max}(t) + e_{\min}(t))/2$.
4) Let $i = i + 1$ and define the protomode function (PMF) as $p_i(t) = r(t) - m(t)$, and let $r(t) = p_i(t)$.
5) Repeat steps 1)–4) on PMF $p_i(t)$ until it is an IMF, then record the IMF $\mathrm{imf}_k(t) = p_i(t)$.
6) Let $r(t) = r(t) - \mathrm{imf}_k(t)$, if the extremum point number of $r(t)$ is larger than three, let $k = k + 1, i = 0$, and go to step 1); otherwise, finish the sifting process.

This process gets several IMFs and a residue $r(t)$. So, for any one-dimension discrete signal x, EMD can finally present it with the following representation:

$$x(t) = \sum_{k=1}^{K} \mathrm{imf}_k(t) + r(t). \tag{1}$$

B. IMF Criteria

The second condition for IMF in the above is too rigid to use, so we need to change it for the implementation of EMD. The essence of the change is to make the instantaneous frequency of IMF meaningful. To guarantee that the IMF components have enough physical sense, Huang [1] limits the size of the standard deviation (SD), and SD is computed from two consecutive PMFs as

$$\mathrm{SD} = \sum_{t=0}^{T} \frac{|p_{i-1}(t) - p_i(t)|^2}{p_{i-1}^2(t)}. \tag{2}$$

The SD constraint is a Cauchy-type criterion [5]. Since SD is unrelated to the definition of IMF, the component obtained with this criterion could not be an IMF. As an improvement to

SD criterion, Rilling *et al.* [6] brought forward a 3-threshold criterion. They define three thresholds θ_1, θ_2, and α, and define $a(t) = (e_{\max}(t) - e_{\min}(t))/2$ and $\sigma(t) = |a(t)/m(t)|$. EMD iterates the sifting process until $\sigma(t) < \theta_1$ for fraction $(1 - \alpha)$ of the total time and $\sigma(t) < \theta_2$ for the remaining fraction. The typical values of the thresholds are $\alpha = 0.05, \theta_1 = 0.05$, and $\theta_2 = 0.5$. In the same paper, a local EMD criterion was proposed to overcome the contaminating problem caused by the singular area. The problem of the 3-threshold criterion is that the thresholds do not adapt to the analyzed signal automatically.

Recently, based on the assumption that the components of the analyzed signal are orthogonal mutually, Cheng *et al.* [5] put forward the energy difference tracking (EDT) method. Suppose signal $x(t)$ contains N mutually orthogonal components $\{x_i(t), i = 1, 2, \ldots, N\}$, and the average of $x_i(t)$ is zero, then

$$\int x_i(t) x_j(t)\, dt \approx 0, \quad i \neq j. \tag{3}$$

When EMD decomposes $x(t)$ and obtains an IMF $c_1(t)$, and after $c_1(t)$ has been separated from $x(t)$, the energy variation caused by decomposition is

$$E_{\text{err}} = \left| \int c_1^2(t)\, dt - \int x(t) c_1(t)\, dt \right|. \tag{4}$$

From the Pythagorean Theorem, E_{err} is a measure of the orthogonality of $c_1(t)$ and of $x(t) - c_1(t)$. Hence, we can track E_{err} in sifting process. When E_{err} reaches a certain minimum and the mean envelope values are small enough (use 3-threshold criterion at first to make the mean envelop values small), the current sifting process is completed and comes to the next IMF's iteration. Thus, the obtained IMF component is an orthogonal one of the original signal. However, the EDT criterion cannot pick the correct PMF as IMF when real components of the analyzed signal have strong correlations.

Damerval *et al.* [7] proposed a criterion based on the number of iterations and the number of IMFs for bidimensional EMD. This criterion saves computational cost and has little boundary effect in the sifting process. The number of iterations and IMFs should be selected carefully. Too few sifting steps cannot eliminate the riding waves, and IMFs obtained will dissatisfy the two IMF conditions. On the other hand, too many sifting steps would sometimes obliterate the intrinsic amplitude variations and make the results physically less meaningful.

In addition, none of the before-mentioned criteria uses frequency or phase information of the analyzed signal. So IMFs obtained with those criteria are prone to have a scale-mixing problem and then have no reasonable interpretation.

III. Bandwidth Criterion for IMF

Given a real valued signal $x(t)$, using the Hilbert transform and analytical signal theory, we can obtain a complex valued signal whose real part is equal to $x(t)$ [9]–[12]. The Hilbert transform of signal $x(t)$ is defined by [11]

$$H[x(t)] = \frac{1}{\pi} \int \frac{x(\tau)}{t - \tau}\, d\tau. \tag{5}$$

Due to the possible singularity at $\tau = t$, the integral is to be considered as a Cauchy principal value. The corresponding complex valued signal (analytic signal) is [11]

$$A[x(t)] = x(t) + jH[x(t)]. \tag{6}$$

For this reason, we use a complex valued signal directly hereafter in this section. Given a normalized (energy is 1) signal $z(t) = a(t)e^{j\varphi(t)}$, the instantaneous frequency of $z(t)$ is defined as $\varphi^{'}(t)$. The Wigner distribution of $z(t)$ is defined as [11]

$$P(t, \omega) = \frac{1}{2\pi} \int z^* \left(t - \frac{1}{2}\tau\right) z \left(t + \frac{1}{2}\tau\right) e^{-j\tau\omega}\, d\tau. \tag{7}$$

Then, we have

$$\omega_t = \varphi^{'}(t) = \int \omega P(\omega \,|\, t)\, d\omega \tag{8}$$

where

$$P(\omega \,|\, t) = \frac{P(t, \omega)}{\int P(t, \omega)\, d\omega}. \tag{9}$$

Consider a multicomponent signal comprised of N components

$$z(t) = a(t)e^{j\varphi(t)} = \sum_{m=1}^{N} x_m(t) = \sum_{m=1}^{N} a_m(t) e^{j\varphi_m(t)} \tag{10}$$

where $a_m(t)$ and $\varphi_m(t)$ are the amplitude and phase of the component $x_m(t)$, respectively. Then

$$a^2(t) = \sum_{m=1}^{N} w_m(t) \tag{11}$$

where

$$w_m(t) = \sum_{n=1}^{N} a_m(t) a_n(t) \cos[\varphi_m(t) - \varphi_n(t)]. \tag{12}$$

Calculating the derivative of $z(t)$, from (10)–(12), we can obtain

$$\varphi^{'}(t) = \sum_{m=1}^{N} \frac{w_m(t)}{a^2(t)} \varphi_m^{'}(t) + G(t) \tag{13}$$

where

$$G(t) = \sum_{m=1}^{N} \sum_{n=1}^{N} \frac{a_m^{'}(t) a_n(t)}{a^2(t)} \sin[\varphi_m(t) - \varphi_n(t)]. \tag{14}$$

Equation (13) shows that the instantaneous frequency for the multicomponent signal has two parts: the first part is a weighted average of the instantaneous frequencies of its components, the second part is related to the amplitude values of its components. When $w_m(t) < 0$, the instantaneous frequency of $z(t)$ tends to go beyond the instantaneous frequency range of its components [10]. For this reason, (13) has some apparent paradoxes associated with instantaneous frequency and yields an irrational explanation of IF.

Equation (13) also implies that the signal tends to have a meaningless IF if it is not monocomponent. For the lack of an exact definition of the monocomponent signal by now, Huang [1] uses two IMF conditions to replace the monocomponent requirement. As mentioned before, IMF conditions are not sufficient conditions for reasonable instantaneous frequency. So the IFs of IMFs are usually meaningless because the IMFs are not monocomponent signals.

This paper will show that a better substitution for the monocomponent signal than the two IMF conditions is the local narrowband signal. We call a signal $z(t) = a(t)e^{j\varphi(t)}$ narrowband if $a(t)$ is a bandlimited signal and the highest frequency of $a(t)$

is far less than $\varphi'(t)$. If any little segment of a signal is narrowbanded, then we call the signal local narrowband.

On the other hand, the instantaneous bandwidth of the signal $z(t)$ at time t is [11]

$$B_t = \left| \frac{a'(t)}{a(t)} \right|. \tag{15}$$

The instantaneous bandwidth is a measure of how $P(t,\omega)$ concentrates in the center IF ω_t at time t. When B_t is very small, $z(t)$ is considered as a local narrowband signal and from (9), the IF has perfect physical sense. If $S(\omega)$ is the Fourier transform of $z(t), B$ is the bandwidth of $z(t)$, and the energy of $z(t)$ is 1, from the knowledge of Fourier analysis, we have [11]

$$\begin{aligned} B^2 &= \int (\omega - \langle\omega\rangle)^2 |S(\omega)|^2 d\omega \\ &= \int z^*(t) \left(\frac{1}{j}\frac{d}{dt} - \langle\omega\rangle \right)^2 z(t)\, dt \end{aligned} \tag{16}$$

where

$$\langle\omega\rangle = \int (\omega_t) a^2(t)\, dt = \int \varphi'(t) a^2(t)\, dt. \tag{17}$$

Substituting $z(t) = a(t)e^{j\varphi(t)}$ into (16), we obtain

$$B^2 = B_a^2 + B_f^2 \tag{18}$$

where

$$B_a^2 = \int \left(\frac{a'(t)}{a(t)} \right)^2 a^2(t)\, dt = \int (B_t)^2 a^2(t)\, dt \tag{19}$$

$$B_f^2 = \int (\varphi'(t) - \langle\omega\rangle)^2 a^2(t)\, dt. \tag{20}$$

Equation (18) implies that bandwidth B has two terms B_a and B_f. We call B_f the frequency bandwidth and B_a the amplitude bandwidth. B_a results from the changes of $a(t)$ and is only associated with amplitude modulating. Furthermore, B_a^2 is the weighted sum of $(B_t)^2$. B_f is the result of changes of IF and reflects the consistency of the IF at all time extents. The smaller the B_f, the closer the scale characteristics at different times are, and the slighter the scale-mixing problem is.

In brief, we modify the sifting process as follows.

1) Use a 3-threshold criterion with large thresholds in the sifting process so that PMFs almost satisfy two IMF conditions.
2) Loop the sifting process until B_t is smaller than the threshold (4 in this paper) in the majority (90%) of the time range so that PMF is a local narrowband signal.
3) Continue sifting until we find the minimum of B_f^2.
4) Separate the final PMF as an IMF and deal with the residual signal for other IMFs.

In the implementation of this process, the calculation of $P(t,\omega)$ is unnecessary because we can use (6) to obtain the analytic signal. Then, we can use (15), (17), and (20) to obtain B_t and B_f^2.

As thresholds in step 1) take large values, this process may need fewer calculations than the 3-threshold criterion method. Even some IMFs need more calculation; the other IMFs would need fewer iterations. On all accounts, the number of sifting iterations are adaptive to the analyzed signal automatically.

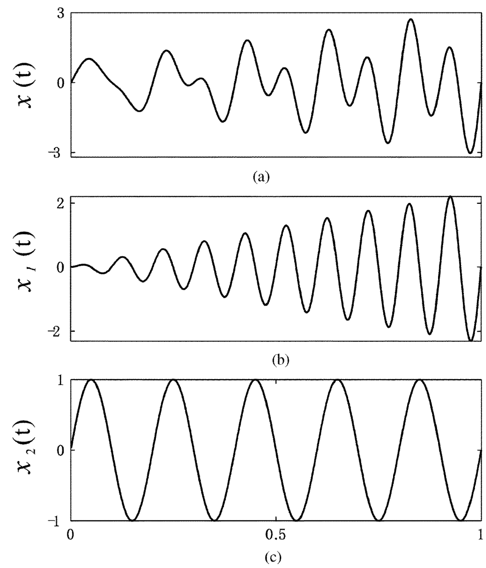

Fig. 1. Simulated signal and its two components.

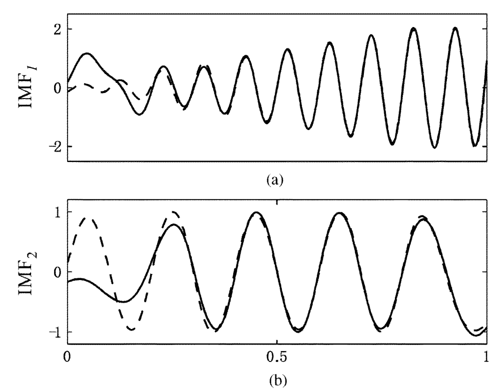

Fig. 2. IMFs of $x(t)$ obtained with 3-threshold criterion (real lines) and bandwidth criterion (lashed lines).

IV. Results

The following illustrate the decomposition results obtained with 3-threshold, EDT, Damerval criterion, and bandwidth criterion, respectively. In order to compare EDT and bandwidth criterion, we choose a signal whose components are almost orthogonal mutually.

As shown in (21) and Fig. 1, the simulated signal $x(t)$ consists of an amplitude-modulated signal and a sine signal

$$\begin{aligned} x_1(t) &= 4\sin(20\pi t)\sin(0.2\pi t) \\ x_2(t) &= \sin(10\pi t) \\ x(t) &= x_1(t) + x_2(t) \end{aligned} \tag{21}$$

where $t \in [0,1]$ and the sampling frequency is 1024 Hz.

The thresholds used in this paper are $\alpha = 0.05, \theta_1 = 0.05$, and $\theta_2 = 0.5$ for 3-threshold and energy difference tracking criterion; $\alpha = 0.1, \theta_1 = 0.1$ and $\theta_2 = 0.5$ for bandwidth criterion; the number of iterations is 100 for the Damerval criterion.

Suppose the IMF corresponding to $x_i(t)$ is $c_i(t)$. We define the error signal between $x_i(t)$ and $c_i(t)$ as

$$D_i(t) = |x_i(t) - c_i(t)| \tag{22}$$

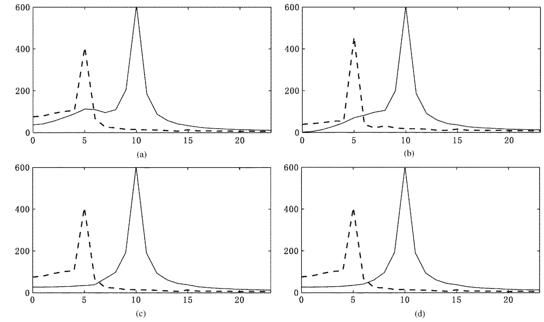

Fig. 3. Fourier spectrums for the IMFs of $x(t)$ with different IMF criteria. Real lines are spectrum for imf_1, and dashed lines are spectrum for imf_2. From left to right and top to bottom: criterion is 3-threshold, Damerval, energy difference tracking, and bandwidth criterion, respectively.

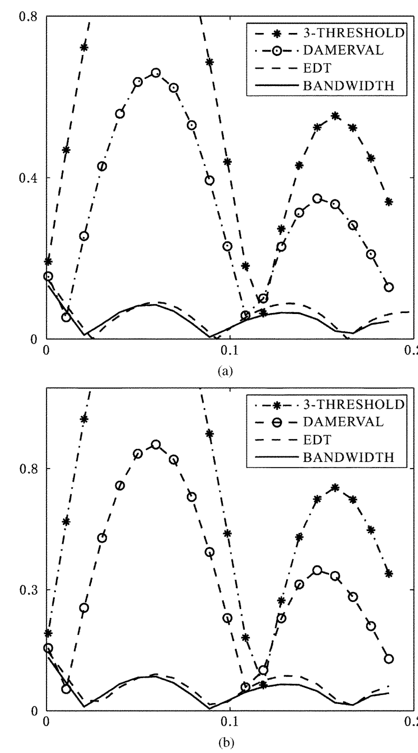

Fig. 4. Part of $D(t)$ with difference criteria. (Top): $D_1(t)$. (Bottom): $D_2(t)$.

and define $E_{\mathrm{dif}}(i)$ as

$$E_{\mathrm{dif}}(i) = \sqrt{\int D_i^2(t)\,dt}. \tag{23}$$

where $D(t)$ describes the performance of EMD. $E_{\mathrm{dif}}(i)$ is the distance between $x_i(t)$ and the corresponding IMF.

Fig. 2 presents the decomposition results of $x(t)$ with 3-threshold criterion and bandwidth criterion. It is clearly illustrated that the foresides of IMFs obtained with 3-threshold criterion are all anamorphic and lose their physical sense. As shown in Fig. 3, the IMFs obtained with 3-threshold and Damerval criterion have a scale-mixing problem because spectrum lines are disordered near 5 Hz. On the contrary, the IMFs obtained with bandwidth have slight scale-mixing problem and are close to the real components. From Fig. 4, we know that $D(t)$ with bandwidth criterion are smaller than $D(t)$ with the other three criteria in the majority of this time range. Table I illustrates that the bandwidth criterion obtains the best approximations to the real components of $x(t)$. In addition, $c_2(t)$ needs only one iteration with bandwidth criterion. This confirms that the numbers of sifting iterations are adaptive to signal automatically.

TABLE I
E_{dif} OF THE SIMULATED SIGNAL WITH DIFFERENT IMF CRITERIA

Criterion	3-Threshold	Damerval Criterion	EDT	Bandwidth
E_{dif} (1)	0.331	0.198	0.110	0.107
E_{dif} (2)	0.254	0.161	0.110	0.105

This example also shows that the bandwidth criterion has some superiority over the EDT criterion even when the real components are nearly orthogonal mutually.

V. CONCLUSION

In this paper, a new IMF criterion is proposed based on two types of bandwidth: instantaneous bandwidth and the frequency bandwidth which is caused only by frequency changes. Compared with criteria that have been considered so far, the bandwidth criterion not only can find the IMFs reflecting the scale and frequency characters of the analyzed signal but also make the IMFs have reasonable meaning. In addition to what has been said, the IMFs obtained with bandwidth criterion have a slighter scale-mixing effect. The criterion proposed here are believed to provide new insights in EMD and IMF, but it cannot resolve the scale-mixing problem fully and requires further study.

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