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Reply to:

Comment on Design of a stable sliding-mode controller for a class of second-order under-actuated systems

The fallacies of a comment by Ma concerning the paper by Wang *et al.* are pointed out.

1 Introduction

Ma [1] claims that the main results in [2] are inconsistent. In this comment, we show that the criticisms are ungrounded and point out Ma's misunderstandings about [2].

2 Response to counterexample given by Ma

Ma claims for the following general two-order system

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(x_1, x_2) \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(X) + b_2(X)u + d_2(t) \end{aligned} \tag{1}$$

The first subsystem $x_1 = x_2$, $x_2 = f_1(x_1, x_2)$ is Lyapunov stable but not asymptotically stable, which contradicts the claims of Theorems 1 and 2 in [2].

The above example is absolutely correct in mathematics. However, if we consider the research background in [2], the counterexample (1) shows that Ma does not understand [2]. The systems adopted in [2] are practical, which exist in mathematics. However, the counterexample (1) is impossible for any practical two-order under-actuated mechanical system.

Spong [3] describes the Lagrangian formulation of the dynamics of a class of under-actuated systems with n-degree-of-freedom and m-control-input ($m < n$) as

$$D(q)\ddot{q} + +C(q, \dot{q})\dot{q} + g(q) = B(q)\tau \tag{2}$$

Here, $q \in R^n$ is the vector of generalised coordinates and $\tau \in R^m$ is the generalised control input. Let $n = 2$ and $m = 1$ in (2). Equation (2) will stand for the dynamics of the class adopted in [2], which can be depicted by

$$\begin{aligned} d_{11}\ddot{q}_1 + d_{12}\ddot{q}_2 + h_1(q_1, \dot{q}_1, q_2, \dot{q}_2) + \varphi_1(q_1, q_2) &= 0 \\ d_{12}\ddot{q}_1 + d_{22}\ddot{q}_2 + h_2(q_1, \dot{q}_1, q_2, \dot{q}_2) + \varphi_2(q_1, q_2) &= b(q_1, q_2)\tau \end{aligned} \tag{3}$$

Here, $q_1 \in R^1$, $q_2 \in R^1$ and $\tau \in R^1$.

Let $x_1 = q_1$, $x_2 = \dot{q}_1$, $x_3 = q_2$ and $x_4 = \dot{q}_2$ in (3). We could have the following state equation, which is given as (7) in [2]

$$\begin{aligned} \dot{x}_1 &= x_2 \\ \dot{x}_2 &= f_1(X) + b_1(X)u \\ \dot{x}_3 &= x_4 \\ \dot{x}_4 &= f_2(X) + b_2(X)u \end{aligned} \tag{4}$$

Here, f_1, f_2, b_1 and b_2 are the functions of the state vector $X = [x_1, x_2, x_3, x_4]$, which means that the two subsystems couple each other. Therefore the above counterexample (1) with the first subsystem decoupling from the second one is not at all the class focused by Wang *et al.* in [2].

By the way, based on Olfati-Saber's systematic method [4] to transform an under-actuated system into a normal form,

(4) could be transformed as

$$\begin{aligned}\dot{x}_1 &= x_2\\ \dot{x}_2 &= p_1(X)\\ \dot{x}_3 &= x_4\\ \dot{x}_4 &= p_2(X) + q_2(X)u\end{aligned} \tag{5}$$

Here, p_1, p_2 and q_2 are the functions of the state vector X. Notice that (5) is again different from (1). The two subsystems couple each other in (5) by the state vector X. In (1), however, the first subsystem decouples from the second one. Furthermore, Xu and Özgüner [5] indicate that the class in [2] with the form (5) could be asymptotically stabilised.

In a word, any theory has its own bound. It is not adequate that Ma takes a special case as the counterexample that is out of the class of the two-order under-actuated systems in [2].

3 Response to other comments

Ma also claims that the proposed control law in [2] can only make either the surface S_1 or S_2 asymptotically stable. In fact, there only exist two-layer sliding surfaces, namely the first layer sliding surfaces s_1 and s_2 and the second layer sliding surface S_1. Equation (26) in [2] constructs S_1 and S_2 just for proving that the coefficients α and β do not influence the system stability. But this does not mean that there exist S_1 and S_2 at the same time in a control process.

Further, Ma's claims [1] implicitly use the assumption $\dot{\alpha} = 0$ which means α is a constant and that this case contradicts Theorem 2. From the viewpoint of variable structure control, system structure is variable in order that we could have (15) as long as α is piecewise smooth. Consequently, Theorem 2 and (15) in [2] do not contradict each other at all. Furthermore, Theorem 2 proposes the idea that soft switch of the coefficient α substitutes hard switch of the control input u, which provides a way to decrease the chattering phenomena of the control input u.

4 Conclusion

Owing to the misunderstandings about under-actuated systems and sliding-mode control, Ma in [1] mistakes the results of Wang *et al.* [2] as fallacies. The criticisms by Ma are false and ungrounded. The control method in [2] enriches sliding-mode control theory and provides a useful tool for control design of the two-order under-actuated systems.

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D. Qian[1] J. Yi[1] W. Wang[2]

[1]Laboratory of Complex Systems and Intelligence Science, Institute of Automation, Chinese Academy of Sciences, Beijing 100190, People's Republic of China

[2]Department of Quality Management, China Astronautics Standards Institute, People's Republic of China
E-mail: dianwei.qian@ia.ac.cn

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