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Stabilization control of series-type double inverted pendulum systems using the SIRMs dynamically connected fuzzy inference model

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Abstract

A new fuzzy controller for stabilizing series-type double inverted pendulum systems is proposed based on the SIRMs (Single Input Rule Modules) dynamically connected fuzzy inference model. The controller deals with six input items. Each input item is provided with a SIRM and a dynamic importance degree (DID). The SIRM and the DID are set up such that the angular control of the upper pendulum takes the highest priority order over the angular control of the lower pendulum and the position control of the cart when the relative angle of the upper pendulum is big. By using the SIRMs and the DIDs, the control priority orders are automatically adjusted according to control situations. Simulation results show that the controller stabilizes series-type double inverted pendulum systems of different parameter values in about 10.0 s for a wide range of the initial angles. © 2001 Elsevier Science Ltd. All rights reserved.

Keywords: Dynamic importance degree; Expert system; Fuzzy control; Intelligent system; Double inverted pendulum; SIRM (Single Input Rule Module); Stabilization

1. Introduction

In a series-type double inverted pendulum system discussed here, two pendulums are linked and the lower pendulum is hinged on a cart. Since the upright state of the two pendulums is an unstable equilibrium point, the two pendulums will fall down without control if one of the pendulums does not stand up. Stabilization control of series-type double inverted pendulum systems is to balance the two pendulums upright and put the cart to a specified position by moving the cart right and left. Because all the angular control of the upper pendulum and the angular control of the lower pendulum and the position control of the cart must be done by only one manipulated variable, this is a very difficult control problem.

To stabilize a series-type double inverted pendulum on an inclined rail, Furuta et al. [1] designed an optimal regulator based on state space theory. The approach had to linearize the originally nonlinear mathematical model and the initial angles of the two pendulums were both limited within $\pm 5°$, which was required in linearization. Guo etc. [2] built a Takagi–Sugeno style fuzzy controller, and the consequent part of each fuzzy rule was a linear state feedback gain vector which had seven parameters to determine. Because the maximum angles of the two pendulums in transient phase became as about four times big as their initial angles, the initial angles of the two pendulums were limited within $\pm 10°$. Terano etc. [3] constructed a three-mode fuzzy controller of four input items based on the behavior of a skilled operator. The fuzzy controller only balanced the two pendulums and did not take the position control of the cart into consideration. Muchammad etc. [4] built a two-stage fuzzy controller to stabilize a series-type double inverted pendulum system. At the first stage, the force necessary for controlling each pendulum was inferred separately. At the second stage, the actual force to be added to the cart was inferred from the two forces obtained at the first stage. To realize the position control of the cart, however, a virtual target angle had to be inferred and then imbedded into the angles of the two pendulums. By training a layered neural network using reinforcement learning, Riedmiller [5] stabilized a series-type double inverted pendulum system with some offset left in the cart position. Because of using a neural network, the control architecture

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became a black box and explicit control knowledge representation was impossible.

In stabilization control of a series-type double inverted pendulum system, six state variables (input items) have to be taken into consideration in order to cover the angular controls of the two pendulums and the position control of the cart. Because the angular controls of the two pendulums should be done first and the position control of the cart should be done after the two pendulums are almost balanced upright, the priority orders of the three controls have to be discriminated clearly. Although state feedback control theory like a regulator can solve such a problem theoretically, a linear mathematical model is necessary and system parameters must be obtained accurately. Furthermore, the control design to determine control parameters is still a hard task even for control experts. On the other hand, a fuzzy controller based on fuzzy inference model can express experts' knowledge in fuzzy rule fashion and work even without mathematical model. However, the conventional non Takagi–Sugeno style fuzzy inference model, which puts all of the input items into the antecedent part of each fuzzy rule, needs many fuzzy rules and has poor ability to express the control priority orders. As a new approach, the SIRMs dynamically connected fuzzy inference model [6,7] assigns each input item with a SIRM (Single Input Rule Module) and a dynamic importance degree (DID). Because the input items can be processed dispersedly by the SIRMs and the control priority orders can be represented definitely by the DIDs, the model has been successfully applied to trajectory tracking control [8] and stabilization control of single inverted pendulum systems [9].

In this paper, a new fuzzy controller for stabilizing series-type double inverted pendulum systems is proposed based on the SIRMs dynamically connected fuzzy inference model. To stabilize series-type double inverted pendulum systems, the proposed fuzzy controller handles six input items and one output item. The SIRMs and the DIDs are set up such that the angular control of the upper pendulum takes the highest priority order over the angular control of the lower pendulum and the position control of the cart. By using the SIRMs and the DIDs, the angular controls of the two pendulums and the position control of the cart are done in parallel, and the priority orders of the three controls are automatically adjusted according to control situations. The simulation results show that with a simple and intuitively understandable structure, the proposed fuzzy controller can completely stabilize series-type double inverted pendulum systems of different parameter values.

2. SIRMS dynamically connected fuzzy inference model

For systems of n input items and 1 output item, the SIRMs dynamically connected fuzzy inference model first defines a Single Input Rule Module (SIRM) separately for each input item as:

$$\text{SIRM-}i: \qquad \{R_i^j:\ \text{if } x_i = A_i^j \text{ then } f_i = C_i^j\}_{j=1}^{m_i}, \tag{1}$$

where SIRM-i denotes the SIRM of the ith input item, and R_i^j is the jth rule in the SIRM-i. The ith input item x_i is the only variable in the antecedent part, and the consequent variable f_i is an intermediate variable corresponding to the output item f. A_i^j and C_i^j are the membership functions of x_i and f_i in the jth rule of the SIRM-i. Further, $i = 1, 2, \ldots, n$ is the index number of the SIRMs, and $j = 1, 2, \ldots, m_i$ is the index number of the rules in the SIRM-i.

If the simplified fuzzy reasoning method [10] is used and the consequent membership function C_i^j is a real number, then the inference result f_i^0 of the consequent variable f_i of the SIRM-i for x_i is given by:

$$f_i^0 = \frac{\sum_{j=1}^{m_i} A_i^j(x_i) C_i^j}{\sum_{j=1}^{m_i} A_i^j(x_i)} \tag{2}$$

To express clearly the different role of each input item in system performance, the SIRMs dynamically connected fuzzy inference model further defines a DID w_i^{D} independently for each input item x_i as:

$$w_i^{\mathrm{D}} = w_i + B_i \Delta w_i^0 \tag{3}$$

The base value w_i guarantees the necessary function of the corresponding input item through a control process. The dynamic value, defined as the product of the breadth B_i and the inference result Δw_i^0 of the dynamic variable Δw_i, plays a role in tuning the degree of the influence of the input item on system performance according to control situations. The base value and the breadth are control parameters, and the dynamic variable is described by fuzzy rules.

After each DID w_i^{D} and the fuzzy inference result f_i^0 of each SIRM are calculated, the SIRMs dynamically connected fuzzy inference model then obtains the output value of the output item f by:

$$f = \sum_{i=1}^{n} w_i^{\mathrm{D}} f_i^0, \tag{4}$$

as the summation of the products of the fuzzy inference result of each SIRM and its DID for all the input items.

To apply the SIRMs dynamically connected fuzzy inference model, therefore, one has to set up the SIRM, the fuzzy rules of the dynamic variable, and the control parameters for all the input items.

3. Series-type double inverted pendulum system

The series-type double inverted pendulum system considered here is shown in Fig. 1. The system consists of

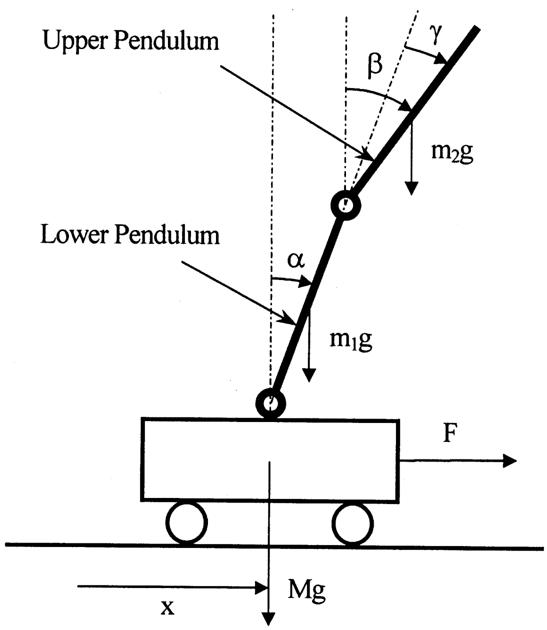

Fig. 1. Configuration of series-type double inverted pendulum system.

a straight-line rail, a cart moving on the rail, a lower pendulum, which is hinged on the cart, and an upper pendulum, which is linked with the other end of the lower pendulum. In the same vertical plane with the rail, the lower pendulum can rotate around the pivot and the upper pendulum can rotate around the linkage.

Here, the parameters M, m_1, m_2 in the unit [kg] are separately the masses of the cart, the lower pendulum and the upper pendulum. The parameter $g = 9.8$ (m/s^2) is the gravity acceleration. Suppose the mass of each pendulum is distributed uniformly. The full length of the lower pendulum and the length from the gravity center of the lower pendulum to the pivot are denoted as L_1, l_1 in the unit (m), and $L_1 = 2l_1$. The length of the gravity center of the upper pendulum to the linkage is denoted as l_2 in the unit (m).

The position of the cart from the rail origin is denoted as x, and is positive when the cart locates on the right side of the rail origin. The angle of the lower pendulum and the angle of the upper pendulum both from upright position are denoted as α and β, which positive directions correspond to clockwise direction. The driving force applied horizontally to the cart is denoted as F in the unit (N), and is positive if pushing the cart toward right direction.

Given that no friction exists in the pendulum system, then the dynamic equation of such a series-type double inverted pendulum system is obtained from Lagrange's equation of motion as:

$$\begin{cases} a_{11}\ddot{x} + a_{12}\ddot{\alpha} + a_{13}\ddot{\beta} = b_1 \\ a_{21}\ddot{x} + a_{22}\ddot{\alpha} + a_{23}\ddot{\beta} = b_2 \,, \\ a_{31}\ddot{x} + a_{32}\ddot{\alpha} + a_{33}\ddot{\beta} = b_3 \end{cases} \tag{5}$$

where the coefficients are given by:

$$\begin{cases} a_{11} = M + m_1 + m_2 \\ a_{22} = 4m_1 l_1^2/3 + m_2 L_1^2 \\ a_{33} = 4m_2 l_2^2/3 \\ a_{12} = a_{21} = (m_1 l_1 + m_2 L_1)\cos\alpha \\ a_{13} = a_{31} = m_2 l_2 \cos\beta \\ a_{23} = a_{32} = m_2 L_1 l_2 \cos(\alpha - \beta) \end{cases} \tag{6}$$

and

$$\begin{cases} b_1 = F + (m_1 l_1 + m_2 L_1)\dot{\alpha}^2 \sin\alpha + m_2 l_2 \dot{\beta}^2 \sin\beta \\ b_2 = (m_1 l_1 + m_2 L_1) g \sin\alpha - m_2 L_1 l_2 \dot{\beta}^2 \sin(\alpha - \beta) \,. \\ b_3 = m_2 l_2 g \sin\beta + m_2 L_1 l_2 \dot{\alpha}^2 \sin(\alpha - \beta) \end{cases} \tag{7}$$

The position x and velocity $\dot{x}$ of the cart, the angle α and angular velocity $\dot{\alpha}$ of the lower pendulum, the angle β and angular velocity $\dot{\beta}$ of the upper pendulum are the state variables.

Note that in Eqs. (5)–(7), the unit of each angle is radian, and the unit of each angular velocity is radian per second. In the following fuzzy inference for convenience, however, the unit of each angle is changed to degree, and the unit of each angular velocity is changed to degree per second.

4. Stabilization fuzzy controller design

In order to stabilize the series-type double inverted pendulum system, as well known from intuition and experience, first the two pendulums should be controlled to the same line, and then the aligned two pendulums are balanced upright, and finally the cart is moved to a specified position. Given the angle of the upper pendulum relative to the lower pendulum as $\gamma(\gamma = \beta - \alpha)$. Then, $\gamma = 0°$ means that the two pendulums are aligned and have the same angle values. Here, the relative angle γ and angular velocity $\dot{\gamma}$ of the upper pendulum, the angle α and angular velocity $\dot{\alpha}$ of the lower pendulum, the position x and velocity $\dot{x}$ of the cart are selected in this order as the input items x_i ($i = 1, 2, \ldots, 6$) after normalization by their own scaling factors. The driving force F after normalization by its scaling factor is chosen as the output item f.

Without losing generality, the rail origin is selected as the desired position of the cart. Then, if the values of the six input items all become zeros, the stabilization control of the series-type double inverted pendulum system is achieved. Therefore, stabilizing the series-type double inverted pendulum system is to design a regulator to converge the values of the six input items to zeros. Here, the fuzzy controller with the six input items and the output item is designed based on the SIRMs dynamically connected fuzzy inference model.

4.1. Setting the SIRMs

It is understood from experience that if the cart moves positively toward right direction, the lower pendulum will rotate counterclockwise and the upper pendulum will rotate clockwise. Therefore in case of positive values of the angle γ and the angular velocity $\dot{\gamma}$, if positive driving force is added to move the cart right, the lower pendulum rotates counterclockwise and the upper pendulum rotates clockwise, causing the angle γ and the angular velocity $\dot{\gamma}$ to increase further. Since the result goes against the stabilization control purpose, it is necessary in this case to move the cart left by negative driving force so that the lower pendulum rotates clockwise and the upper pendulum rotates counterclockwise. Fig. 2 shows the relationship of the applied driving force and the resultant rotation directions of the two pendulums. The solid line to the cart denotes the applied force, and the dotted lines on the pendulums denote the resultant rotation directions. In the same way, in case of negative values of the angle γ and the angular velocity $\dot{\gamma}$, positive driving force is necessary to move the cart toward right direction such that the lower pendulum rotates counterclockwise and the upper pendulum rotates clockwise. By this setting, the angle γ and the angular velocity $\dot{\gamma}$ tend to become zeros, and the two pendulums are controlled to the same line.

Suppose the two pendulums are already aligned. In case of positive values of the angle α and the angular velocity $\dot{\alpha}$ of the lower pendulum, if positive driving force is applied to move the cart right, the lower pendulum will rotate counterclockwise toward upright position. At the same time, however, the upper pendulum will rotate clockwise, making its angle from upright position increase furthermore. As well known intuitively, the stabilization control will become more difficult if the two pendulums incline to the same side and the upper pendulum has a bigger angle than the lower pendulum. In this case as shown in Fig. 3, therefore, it is important to add negative driving force to the cart so that the upper pendulum rotates counterclockwise toward upright position while the lower pendulum falls down clockwise a little more. If the angle α and the angular velocity $\dot{\alpha}$ of the lower pendulum are negative, positive driving force has to be applied to the cart in the same reason. As the result, the angle γ and the angular velocity $\dot{\gamma}$ will have different signs from the angle α and the angular velocity $\dot{\alpha}$ of the lower pendulum. Then through the control of the angle γ and the angular velocity $\dot{\gamma}$, the two pendulums will rotate toward upright position from opposite directions, and both the angle γ and the angle α will get smaller. Consequently, the two pendulums tend to be balanced upright.

Further suppose the two pendulums already stand up upright. When the position x and the velocity $\dot{x}$ of the cart are positive, adding negative driving force to the cart toward left direction will make the lower pendulum rotate clockwise and the upper pendulum rotate counterclockwise. Because the angle γ and the angular velocity $\dot{\gamma}$ become negative, the control of the angle γ and the angular velocity $\dot{\gamma}$ generates positive driving force to move the cart toward right direction. Then the lower pendulum rotates counterclockwise and the upper pendulum rotates clockwise. Because the cart moving directly affects the lower pendulum, the lower pendulum rotates faster than the upper pendulum. As the result, the angle of the lower pendulum becomes negative, and the angle γ and the angular velocity $\dot{\gamma}$ become positive. Therefore, the control of the angle γ and the angular velocity $\dot{\gamma}$ outputs negative driving force to put the cart back to the rail origin and at the same time balance the two pendulums upright. This is also true for negative position x and negative velocity $\dot{x}$ of the cart.

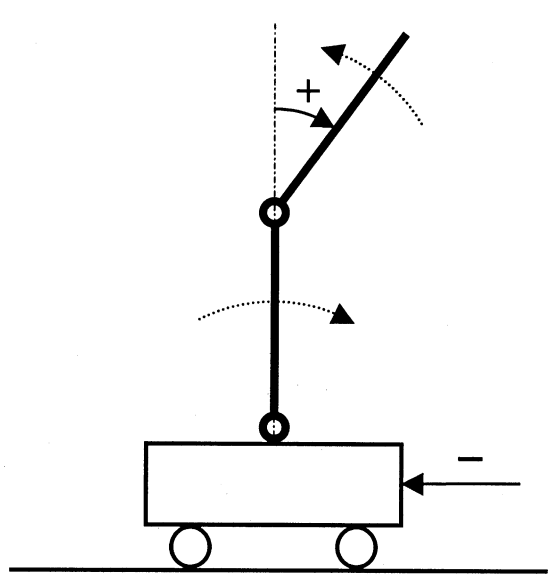

Fig. 2. Relationship between positive relative angle, negative driving force, and resultant pendulum rotations.

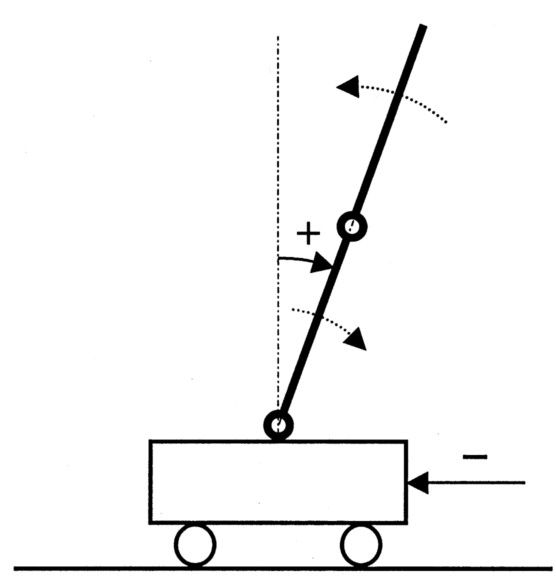

Fig. 3. Relationship between positive lower pendulum angle, negative driving force, and resultant pendulum rotations.

Table 1
SIRM setting for each of the six input items

Antecedent variable x_i	Consequent variable f_i
NB	1.0
ZO	0.0
PB	-1.0

Therefore, the SIRMs of the six input items can all be set up in Table 1. Here, the membership functions of each antecedent variable are defined in Fig. 4 as triangle or trapezoids. The consequent variable f_i $(i = 1, 2, \ldots, 6)$ of each SIRM is an intermediate variable all corresponding to the output item f of the fuzzy controller. Because the simplified reasoning method is adopted here, real numbers are assigned as singleton-type membership functions to the consequent variable of each SIRM.

4.2. Control priorities and the DIDs

As can be seen from Table 1, all the SIRMs have the same rule setting and infer directly the output item of the fuzzy controller. Based on Table 1, the angular control of the upper pendulum decreases the relative angle of the upper pendulum, while the angular control of the lower pendulum and the position control of the cart both serve to make the angular control of the upper pendulum feasible. This implies that balancing the lower pendulum and moving the cart to the rail origin have to be realized through the angular control of the upper pendulum. In order to achieve the stabilization control, then the priority orders of the three controls are considered.

If the absolute value of the angle γ is big, it should be reduced by immediate control action. Otherwise because of the influence of the pendulum weight, the upper pendulum will rotate further toward the same direction, causing the angle γ to get even bigger. If the absolute value of the angle γ is big enough, it will become impossible to stand up the upper pendulum again.

If the lower pendulum takes control priority over the upper pendulum, the angular control of the upper pendulum will become difficult because the two pendulums rotate toward opposite directions and the relative angle γ increases. Moreover, emphasizing the angular control of the lower pendulum based on Table 1 will lead the lower pendulum to fall down because the two SIRMs corresponding to the angle α and the angular velocity $\dot{\alpha}$ of the lower pendulum enlarge the absolute value of the angle α.

If the cart is controlled first before the two pendulums are stood up, the two pendulums will fall down because the lower pendulum rotates toward the opposite direction to the cart moving and the upper pendulum rotates toward the same direction with the cart moving. Therefore, the position control of the cart is permitted only after the two pendulums are almost balanced upright. To keep the balanced state of the two pendulums, it is also necessary to move the cart rather slowly.

In the stabilization control of the series-type double inverted pendulum system, therefore, the angular control of the upper pendulum is the most important. When the angle γ is big, the angular control of the upper pendulum should have the highest priority and the position control of the cart should have the lowest priority. Furthermore, the control priority orders should change with control situations in order to make the angular control of the lower pendulum and the position control of the cart possible.

On the other hand, the DIDs indicate the influence strengths of the input items on system performance and can express explicitly the priority orders. Since the upper pendulum, the lower pendulum, and the cart have two input items each, the control priority orders of the upper pendulum, the lower pendulum, and the cart are represented by the DIDs of its own two input items. The bigger the value of a DID is, the higher the priority order of the corresponding input item becomes.

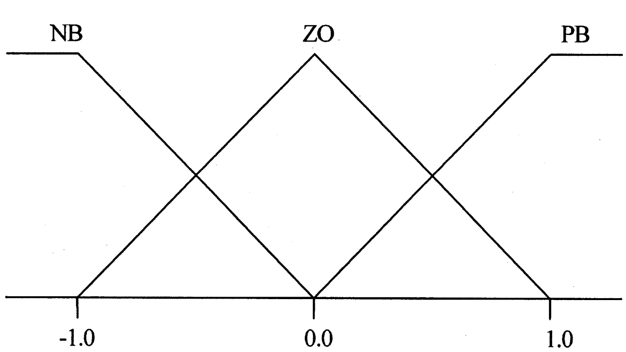

Fig. 4. Membership functions for each SIRM.

4.3. Setting the dynamic variables of the DIDs

The fuzzy rules for the dynamic variables Δw_1 and Δw_2 of the DIDs w_1^{D} and w_2^{D} of the input items x_1 and x_2 corresponding to the angle γ and the angular velocity $\dot{\gamma}$ of the upper pendulum are established in Table 2 by selecting the absolute value of the input item x_1 as the only antecedent variable. Here, the membership functions DS, DM, DB are defined in Fig. 5. In this way, the dynamic variables Δw_1 and Δw_2 always have the same value and adjust separately the DIDs w_1^{D} and w_2^{D} of the upper pendulum in the same direction. When the absolute value of the angle γ is big, the inference results of the dynamic variables Δw_1 and Δw_2 will become big. Therefore, the two DIDs w_1^{D} and w_2^{D} increase so that the angular control of the upper pendulum is

Table 2
Fuzzy rules for the two dynamic variables of the upper pendulum

Antecedent variable $\lvert x_1 \rvert$	Consequent variable Δw_1, Δw_2
DS	0.0
DM	0.5
DB	1.0

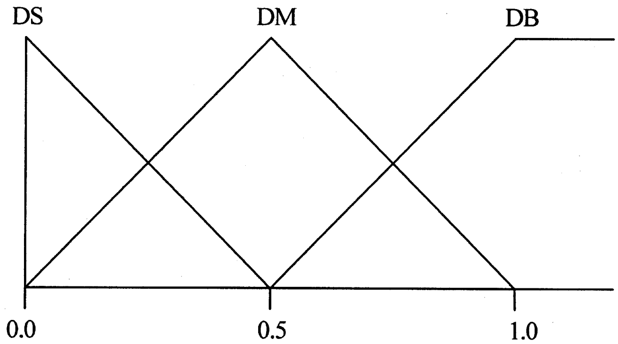

Fig. 5. Membership functions for each dynamic variable.

strengthened. If the absolute value of the angle γ is near zero, the inference results of the dynamic variables Δw_1 and Δw_2 will also become almost zero, and the two DIDs w_1^D and w_2^D will decrease nearly to their base values. As the result, the influence strength of the angular control of the upper pendulum is weakened.

Because the angular control of the upper pendulum has to be done first when the absolute value of the angle γ is big, the fuzzy rules for the dynamic variables Δw_3 and Δw_4 of the DIDs w_3^D and w_4^D of the input items x_3 and x_4 corresponding to the angle α and angular velocity $\dot{\alpha}$ of the lower pendulum are set up in Table 3. Here, the absolute values of the input items x_1 and x_3 corresponding separately to the angle γ and the angle α are chosen as the antecedent variables. In this way, the dynamic variables Δw_3 and Δw_4 always have the same value and adjust separately the DIDs w_3^D and w_4^D of the lower pendulum in the same direction. In Table 3, the real number outputs of the consequent part are set up to 0.0 in those fuzzy rules of $|x_1| = \mathrm{DB}$. By this setting, if the absolute value of the angle γ is not big, then the inference results of the dynamic variables Δw_3 and Δw_4 will increase when the absolute value of the angle α gets big. As the result, the corresponding two DIDs both become large so that the angular control of the lower pendulum is emphasized. When the absolute value of the angle α becomes small, the inference results of the dynamic variables Δw_3 and Δw_4 will decrease. Therefore, the corresponding two DIDs both get small such that the angular control of the lower pendulum has lower priority order.

To do the position control of the cart, the two pendulums should already be almost balanced upright. Therefore, the dynamic variables Δw_5 and Δw_6 of the DIDs w_5^D and w_6^D of the input items x_5 and x_6 corresponding to the position x and velocity $\dot{x}$ of the cart can be described by the fuzzy rules in Table 4. Here, the absolute values of the input items x_1 and x_3 are used as the antecedent variables. In this way, the dynamic variables Δw_5 and Δw_6 always have the same value and adjust separately the DIDs w_5^D and w_6^D of the cart in the same direction. In Table 4, the real number outputs of the consequent part are set up to 0.0 in those fuzzy rules of $|x_1| = \mathrm{DB}$ or $|x_3| = \mathrm{DB}$. By this setting, when the two pendulums are almost stood up, the inference results of the dynamic variables Δw_5 and Δw_6 will become big. Resultantly, the two DIDs of the cart will increase relatively, making the position control of the cart become possible. When one of the two pendulums is still not stood up, the inference results of the dynamic variables Δw_5 and Δw_6 will be small. As the result, the DIDs of the cart decrease so that the position control of the cart loses its control priority.

4.4. Setting the control parameters of the DIDs

Till now, the rule settings of the SIRMs and the dynamic variables of the DIDs have been made clear. As stated above, each DID also has two control parameters, i.e. the base value and the breadth. The rule setting of the dynamic variables only does not guarantee the necessary control priority orders. The control parameters have also to be adequately set up. In order for the angular control of the upper pendulum to take the highest priority order, the sum of the base value and the breadth of each DID of the upper pendulum apparently has to be larger than that of the other DIDs. Further, the sum of the base value and the breadth of each DID of the cart must be the smallest so that the position control of the cart has the lowest priority order.

Here, the system parameters are given in Table 5. The scaling factors of the input items are set up to 15.0°, 100.0°/s, 15.0°, 100.0°/s, 2.4 m, 1.0 m/s, respectively. The scaling factor of the output item is defined as 10 times the total mass of the two pendulums and the cart. Compared with the single inverted pendulum systems [9], the scaling factors of the two pendulum angles are reduced half while the others keep unchanged just because the controllable ranges of the two pendulum angles are narrower.

To tune automatically the control parameters, the random optimization search method [11] is adopted. In each trial, sampling period and total control time are separately fixed

Table 3
Fuzzy rules for the two dynamic variables of the lower pendulum

Δw_3, Δw_4		$\|x_1\|$		
		DS	DM	DB
$\|x_3\|$	DS	0.0	0.0	0.0
	DM	0.5	0.0	0.0
	DB	1.0	0.5	0.0

Table 4
Fuzzy rules for the two dynamic variables of the cart

Δw_5, Δw_6		$\|x_1\|$		
		DS	DM	DB
$\|x_3\|$	DS	1.0	0.5	0.0
	DM	0.5	0.0	0.0
	DB	0.0	0.0	0.0

Table 5
System parameters

System parameter	Value
Lower pendulum mass	0.2 kg
Lower pendulum half-length	0.4 m
Upper pendulum mass	0.1 kg
Upper pendulum half-length	0.2 m
Cart mass	1.0 kg

to 0.01 and 25.0 s. The initial angle of the upper pendulum is set up to 10.0°, while the initial values of the other state variables are all set up to zeros. The base values and the breadths of the DIDs are initially set up to zeros. The random optimization search is run for 7500 trials along such a direction that the total summation of the absolute values of all the state variables and the driving force at each sampling step from the control beginning to the end of the total control time is reduced. If the maximum driving force is given beforehand, the maximum driving force should be reflected in the performance function so that the constraint condition is kept. The base values and the breadths after the random optimization search are shown in Table 6.

As can be seen from Table 6, the control parameters reflect the control priority orders very well although obtained through the random optimization search. For example, the base values and the breadths of the input items x_1 and x_2 of the upper pendulum are separately larger than the base values and the breadths of all the other input items. And the sum of the base value and the breadth of either of the input items x_1 and x_2, i.e. the maximum of the DID, is much larger than that of any of the other input items. Although the breadths of the input items x_3 and x_4 of the lower pendulum are almost as large as the breadths of the input items x_5 and x_6 of the cart, the base values of the input items x_3 and x_4 are much larger than the base values of the input items x_5 and x_6. As the result, the sum of the base value and the breadth of either of the input items x_3 and x_4 is about or more than twice that of either of the input items x_5 and x_6.

4.5. Block diagram of the fuzzy controller

The block diagram of the fuzzy control system for the stabilization control of the series-type double inverted pendulum system is depicted in Fig. 6. The variables γ, $\dot{\gamma}$, α, $\dot{\alpha}$, x, $\dot{x}$ from the pendulum system is fed back and compared with the desired values. Because the desired values are all zeros in the stabilization control, the variables are reversely inputted into the normalizer block. The normalizer block normalizes the variables by their scaling factors each and forms the input items x_i $(i = 1, 2, \ldots, 6)$. Each input item x_i is then guided to the SIRM-i block, where the fuzzy inference of the SIRM corresponding to the input item x_i is done. The DID-1 block and DID-2 block take the absolute value of the input item x_1 as their antecedent variable, while the other DID blocks take both the absolute values of the input items x_1 and x_3 as their antecedent variables. The DID-*i* block calculates the value of the DID of the input item x_i. After the output of each SIRM-*i* block is multiplied by the output of the DID-*i* block, summing them for all the input items gives the output value of the output item f of the fuzzy controller. The Output Scaling Factor (OSF) block finally multiplies the output of the fuzzy controller by the scaling factor of the output item to determine the actual driving force F to the cart.

Although there are six SIRM blocks and six DID blocks in the fuzzy controller, each block executes simple processing only. Furthermore, each SIRM-*i* block infers an intermediate variable related with the output item of the fuzzy controller, and each DID-*i* block tunes a DID according to control situations. The product of each SIRM-*i* block and its corresponding DID-*i* block is actually a part of the output item of the fuzzy controller. Therefore, the angular control of the upper pendulum, the angular control of the lower pendulum, and the position control of the cart are performed completely in parallel. The three controls cooperate and compete with each other in determining the output of the fuzzy controller. Which plays the leading role then depends on control situations besides the proposed structure and the control parameters.

If the absolute value of the relative angle γ is big, the angular control of the upper pendulum will take the highest priority because the two DIDs of the upper pendulum become the biggest. Resultantly, the upper pendulum will rotate toward such a direction that the angle γ gets small. If the absolute value of the angle γ is small and the absolute value of the angle α of the lower pendulum is big, the two DIDs of the lower pendulum increase while the two DIDs of the upper pendulum decrease. At the same time, the inference result of the SIRM corresponding to the angle α becomes large, while the inference result of the SIRM corresponding to the angle γ becomes small. As the result, the contribution of the input items x_3 and x_4 in Eq. (4) will exceed that of the input items x_1 and x_2 so that the angular control of the lower pendulum becomes the main. If both the absolute values of the angles γ and α are small, the inference results of the two SIRMs corresponding to the angles γ and α will get small. At the same time, the two DIDs of the cart increase, while the DIDs of the two pendulums decrease. Consequently, the contribution of the input

Table 6
Control parameters of the DIDs

Input item	Base value	Breadth
x_1	2.9277	1.9227
x_2	2.1918	1.0117
x_3	0.4186	0.2543
x_4	2.1629	0.0663
x_5	0.0590	0.2930
x_6	0.1711	0.1154

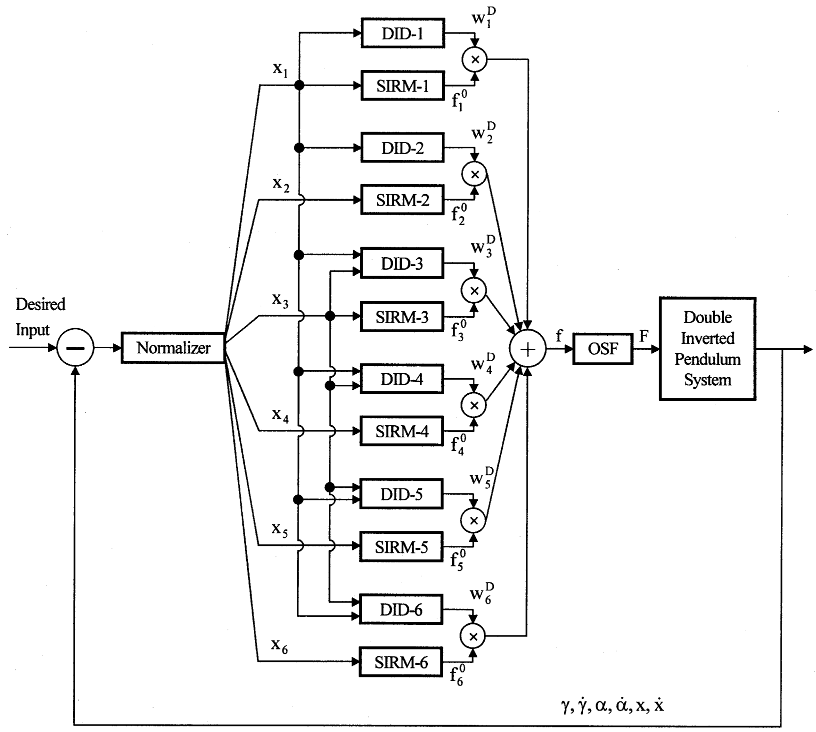

Fig. 6. Block diagram of the fuzzy control system.

items x_5 and x_6 in Eq. (4) increases relatively, making it possible to start the position control of the cart. By using the SIRMs and adjusting the DIDs according to control situations, therefore, the angular control of the upper pendulum, the angular control of the lower pendulum, and the position control of the cart are switched automatically. In this way, the stabilization control of the series-type double inverted pendulum system is achieved.

5. Stabilization control simulations

To verify the effectiveness of the proposed stabilization fuzzy controller, control simulation is done first for the series-type double inverted pendulum system used in the random optimization search. In the simulations, the scaling factors of the input items and the control parameters are all fixed.

Fig. 7 shows the control results in order when the initial angles α and β of the two pendulums are separately set up to 15.0 and 10.0, 15.0 and 15.0, 15.0 and 20.0°, while the initial values of the other state variables are all reset to zeros. The left axis and the right axis separately represent the angle of the pendulums in degree and the position of the cart in meter. The values in P (0.20, 0.40, 0.10, 0.20, 1.00, 0.01) mean in this order the mass and half-length of the lower pendulum, the mass and half-length of the upper pendulum, the cart mass, and the sampling period. The values in S (15.0, 0.0, 10.0, 0.0, 0.0, 0.0) denote the initial values of the angle and angular velocity of the lower pendulum, the angle and angular velocity of the upper pendulum, the position and velocity of the cart, respectively.

In Fig. 7(a), the relative angle γ (= $-5.0°$) of the upper pendulum has a different sign from that of the lower pendulum at control beginning. Since the absolute value of the angle α is three times as large as the absolute value of the angle γ, the inference result of the SIRM of the input item x_3 is also three times as large as that of the SIRM of the input item x_1 from the SIRM setting. Because the DID of the input item x_1 is about six times as large as that of the input item x_3 at control beginning, however, the input item x_1 totally contributes more to the output item in Eq. (4). Therefore, the angular control of the upper pendulum is started first by moving the cart right. As the result, the lower pendulum rotates counterclockwise and the upper pendulum rotates clockwise. When the relative angle γ becomes almost zero, the angular control of the lower pendulum becomes the main part in Eq. (4). Since the angle of the lower pendulum is positive then, negative driving force is added to the cart. However, the cart still moves toward right direction because of inertia. When the relative angle γ gets positive enough, the angular control of the upper pendulum takes priority again and negative driving force is continuously applied to the cart. Then the cart begins to move back toward the rail origin and two pendulums are aligned to the same line. Finally, the two pendulums are balanced upright and the cart is put back to the rail origin by fine switching of the three controls.

In Fig. 7(b), since at control beginning the relative angle γ of the upper pendulum is just 0.0° and the angle α of the

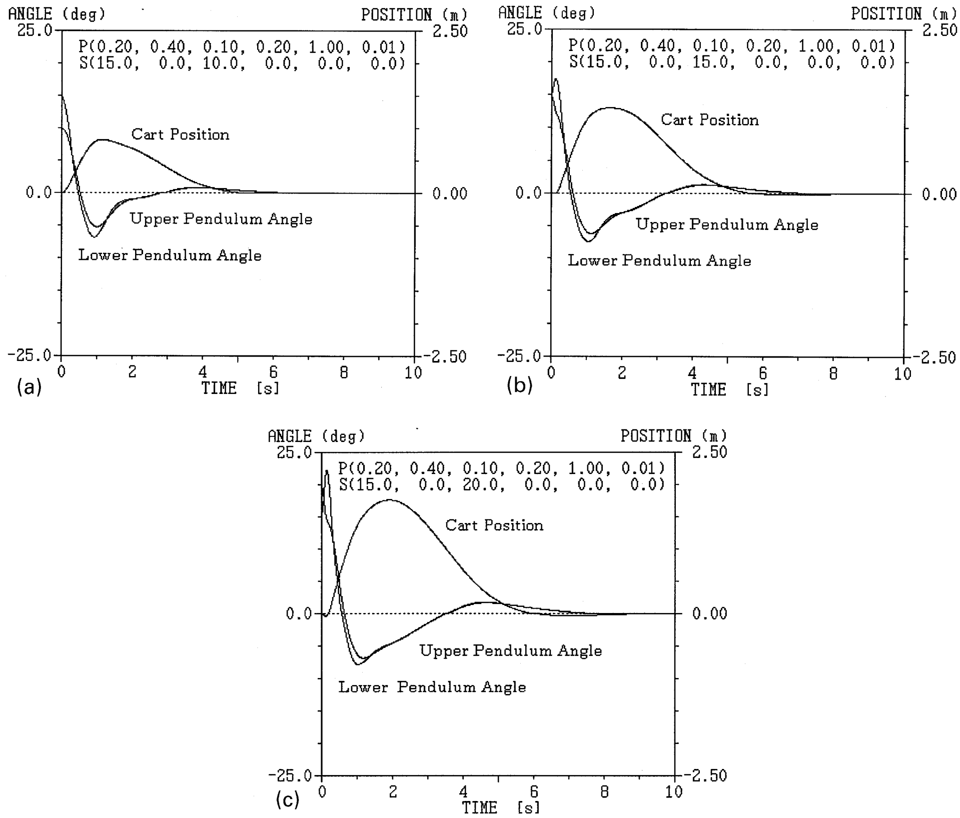

Fig. 7. Control results of pendulum lengths 0.8 and 0.4 m. (a) Initial angles 15.0 and 10.0°; (b) Initial angles 15.0 and 15.0° (c) Initial angles 15.0 and 20.0°.

lower pendulum is 15.0°, the angular control of the lower pendulum is done first by moving the cart to about -0.01 m although it is hard to read directly from the graph. Resultantly, the lower pendulum falls down further to about 17.0°, and the upper pendulum rotates counterclockwise to about 12.0°. The situation from then resembles the start state of Fig. 7(a), and the pendulum system is stabilized in the similar way.

In Fig. 7(c), since the relative angle γ $(= 5.0°)$ of the upper pendulum has the same sign with that of the lower pendulum at control beginning, the angular control of the upper pendulum and the angular control of the lower pendulum both take negative driving force at first. After the cart is moved to about -0.05 m, the angle α of the lower pendulum increases to about 22.0°, while the angle β of the upper pendulum decreases to about 15.0°. Then the pendulum system is stabilized in the way similar to Fig. 7(a).

Although the three sets of the initial state are different from that learnt in the random optimization search, the pendulum system is stabilized smoothly by using the proposed fuzzy controller. Defining complete stabilization time as the time interval from control beginning to such a state that all the state variables just converge separately to 0.1°, 0.1°/s, 0.1°, 0.1°/s, 0.01 m, 0.01 m/s, the complete stabilization time for the three sets of the initial state are about 7.19, 8.46, 9.27 s, respectively. Moreover, the maximum driving forces in the three examples are separately about 8.0, 12.0, 25.0 N, although the time response of the driving force is not depicted for lack of space.

Fig. 8 shows the stabilization domain of the initial angles of the two pendulums, for which the proposed fuzzy controller

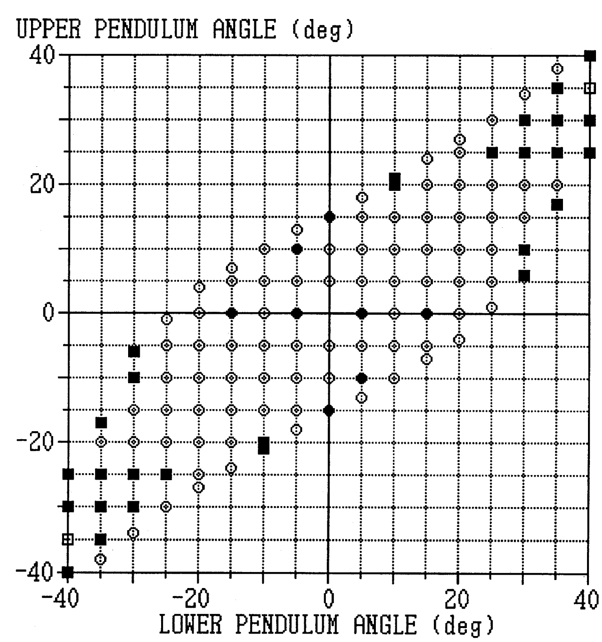

Fig. 8. Stabilization domain of pendulum lengths 0.8 and 0.4 m.

can stabilize the pendulum system. Here, the horizontal axis and the vertical axis stand separately for the initial angles of the lower pendulum and the upper pendulum in degree. The initial angles of the two pendulums both are selected every 5.0° from −40.0 to +40.0°, and the initial values of the other state variables are all fixed to zeros. For such a selected angle of the lower pendulum, the maximum angle of the upper pendulum that can be stabilized is also plotted. The symbols ●, ○, ■, □ mean that the complete stabilization time is within 5.0, 10.0, 15.0 s, or exceeds 15.0 s, respectively. Further, the failure limits of the angle of the lower pendulum, the angle of the upper pendulum, the position of the cart are set up to [−40.0, +40.0°], [−40.0, +40.0°], [−5.0, +5.0 m], respectively. If any of the failure limits is broken off during a simulation, the simulation is regarded as a failure.

As can be seen from Fig. 8, if the initial angle of the lower pendulum is chosen from [−30.0, +30.0°] and the initial angle of the upper pendulum is selected in a range of about 30.0° around the lower pendulum, the pendulum system can be stabilized by the fuzzy controller. For the initial angle of the lower pendulum beyond ±30.0°, if the initial angle of the upper pendulum is selected in a range of more than 15.0° around the lower pendulum, the stabilization control is possible. For most of the controllable sets of the initial angles, the complete stabilization time is within 10.0 s. As the initial angle of the lower pendulum increases, the maximum of the initial angle of the upper pendulum, which can be stabilized, increases slowly. This agrees well with the fact that the stabilization control is difficult or even impossible if the two pendulums incline largely to the same side and the upper pendulum inclines more than the lower pendulum.

Fig. 9 draws a simulation result stabilizing a shorter series-type double inverted pendulum system of $m_1 = 0.1$ kg, $l_1 = 0.1$ m, $m_2 = 0.1$ kg, $l_2 = 0.1$ m, $M = 1.0$ kg for the initial angles 15.0 and 10.0° of the two pendulums.

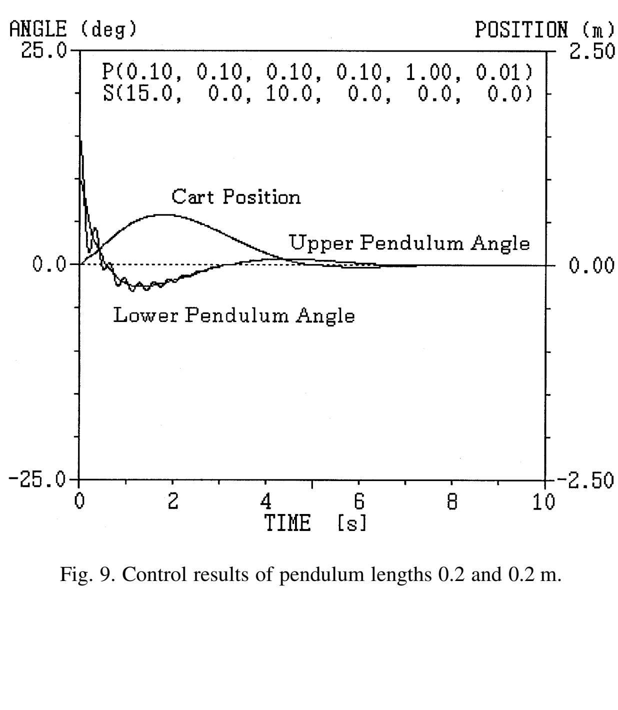

Fig. 9. Control results of pendulum lengths 0.2 and 0.2 m.

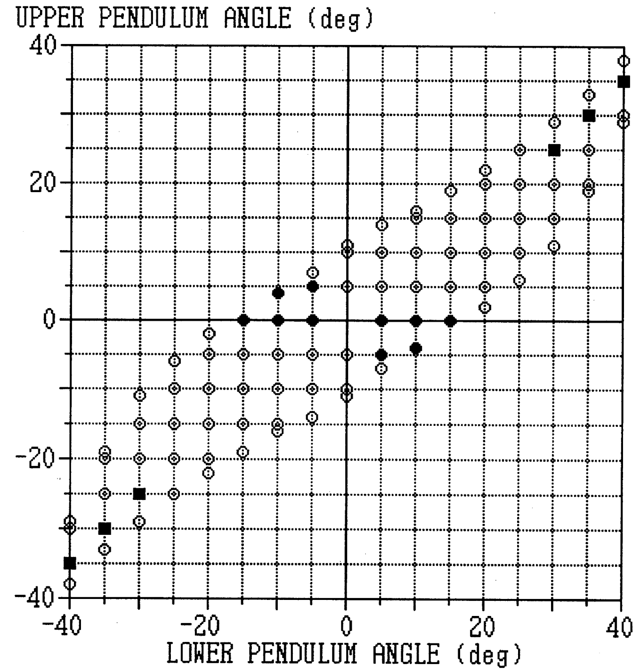

Fig. 10. Stabilization domain of pendulum lengths 0.2 and 0.2 m.

Since the cart moving does not directly affect the angular control of the upper pendulum, the upper pendulum rotates smoothly. On the other hand, because the lower pendulum has high frequency characteristic and undergoes directly the influence of the cart moving, it is liable to vibrate. As the result, in the time response of the lower pendulum, vibration is observed during a period from control beginning to about 3.0 s. After that, the lower pendulum gets almost synchronized with the upper pendulum, and is controlled toward upright position. In this example, the complete stabilization time is about 8.22 s and the maximum driving force is about 9.0 N.

The stabilization domain of the shorter series-type double inverted pendulum system is plotted in Fig. 10. Compared with that of Fig. 8, it is found that the stabilization domain becomes a bit narrow because of the influence of the high frequency characteristic of the lower pendulum. If the lower pendulum initially inclines within [−30.0, +30.0°] and the initial angle of the upper pendulum is chosen in a range of about 20.0° around the lower pendulum, however, the pendulum system can be stabilized by the fuzzy controller. Moreover, for almost all the marked initial states in Fig. 10, the pendulum system can be completely stabilized in 10.0 s.

Figs. 11 and 12 show a stabilization control result and the stabilization domain of a longer series-type double inverted pendulum system, where $m_1 = 0.1$ kg, $l_1 = 0.5$ m, $m_2 = 0.1$ kg, $l_2 = 0.5$ m, $M = 1.0$ kg. In Fig. 11, the initial angles of the two pendulums are both set up to 15.0°. In this example, the pendulum system is completely stabilized in about 10.13 s without vibration, and the maximum driving force is about 17.0 N. From Fig. 12, if the initial angle of the lower pendulum is set up within [−30.0, +30.0°], the initial

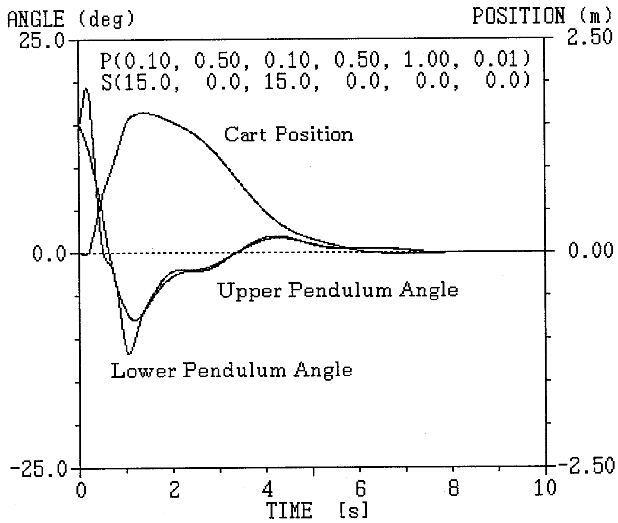

Fig. 11. Control results of pendulum lengths 1.0 and 1.0 m.

angle of the upper pendulum can be selected in a range of more than 25.0° around the lower pendulum. Further, for almost all the initial states plotted in Fig. 12, the pendulum system can be completely stabilized in 15.0 s. By the way, Muchammad etc. [4] stabilized the same pendulum system in more than 40.0 s even for the initial angles 4.0 and 3.0° of the two pendulums, and showed a stabilization domain, which was about 5.0° narrow compared with Fig. 12.

It is found through control simulations that by using the proposed fuzzy controller, most of the series-type double inverted pendulum systems, where the full lengths of the two pendulums are limited between 0.2 and 1.2 m, can be stabilized for a wide range of the initial angles. If the lower

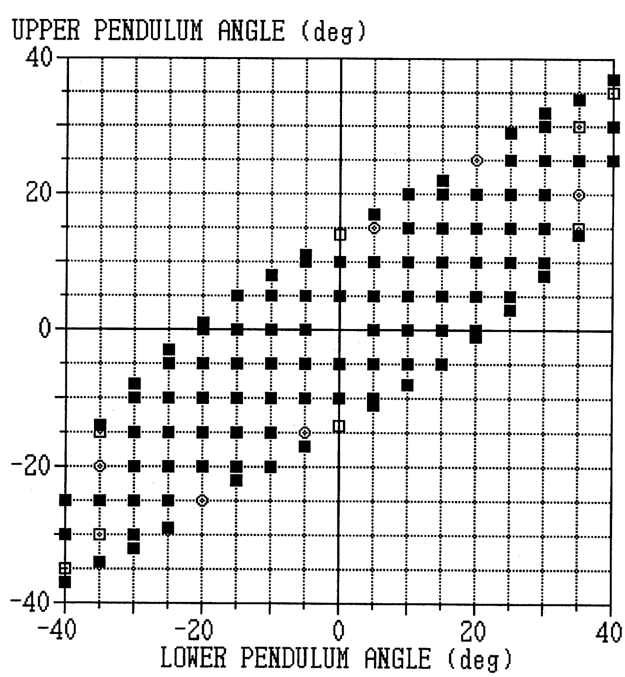

Fig. 12. Stabilization domain of pendulum lengths 1.0 and 1.0 m.

pendulum is rather short, the lower pendulum tends to swing left and right at controlling beginning. In this case, if the sampling period is big, the lower pendulum may fall down because of delay of control action. To weaken the vibration of the lower pendulum, small sampling period is effective. On the other hand, if the lower pendulum is long enough, almost similar stabilization results can be obtained even though the sampling period is set up to 0.02 s. Since the proposed fuzzy controller can stabilize in short time interval the series-type double inverted pendulum systems of different parameter values for a wide range of the initial angles of the two pendulums, the controller can be said to have a high generalization ability.

6. Conclusions

A new fuzzy controller is proposed based on the SIRMs dynamically connected fuzzy inference model for the stabilization control of series-type double inverted pendulum cart systems. The fuzzy controller takes the relative angle and angular velocity of the upper pendulum, the angle and angular velocity of the lower pendulum, and the position and velocity of the cart as the input items, and the driving force as the output item. Each input item is assigned with a SIRM and a DID. The SIRMs and the DIDs are established such that the upper pendulum takes the highest priority order over the angular control of the lower pendulum and the position control of the cart. The fuzzy controller has a simple and intuitively understandable structure, and performs the angular control of the upper pendulum, the angular control of the lower pendulum, the position control of the cart in parallel. By using the SIRMs and the DIDs, the control priority orders are automatically switched according to control situations. Simulation results show that the fuzzy controller has a high generalization ability to completely stabilize series-type double inverted pendulum systems of different parameter values in about 10.0 s for a wide range of the initial angles of the two pendulums. Based on the proposed approach, stabilization control of series-type triple inverted pendulum system is even possible.

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