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Fuzzy Sets and Systems 126 (2002) 105–119

FUZZY sets and systems

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A new fuzzy controller for stabilization of parallel-type double inverted pendulum system

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Received 1 May 2000; received in revised form 14 November 2000; accepted 28 November 2000

Abstract

A new fuzzy controller with 6 input items and 1 output item for stabilizing a parallel-type double inverted pendulum system is presented based on the single input rule modules (SIRMs) dynamically connected fuzzy inference model. Each input item is assigned with a SIRM and a dynamic importance degree. The SIRMs and the dynamic importance degrees are designed such that the angular control of the longer pendulum takes the highest priority over the angular control of the shorter pendulum and the position control of the cart when the angle of the longer pendulum is big. By using the SIRMs and the dynamic importance degrees, the priority orders of the three controls are automatically adjusted according to control situations. The proposed fuzzy controller has a simple and intuitively understandable structure, and executes the three controls entirely in parallel. Simulation results show that the proposed fuzzy controller can stabilize completely a parallel-type double inverted pendulum system within 10.0 s for a wide range of the initial angles of the two pendulums. This is the first result for a fuzzy controller to achieve successfully complete stabilization control of a parallel-type double inverted pendulum system.

Keywords: Fuzzy control; Intelligent system; Inverted pendulum; Single input rule module (SIRM); Stabilization

1. Introduction

As typical unstable nonlinear models, inverted pendulum systems are often used as a benchmark for verifying the effectiveness of a new control method because of the simplicity of the structure. The family of inverted pendulum systems can be classified into single inverted pendulum systems [2,5,7,11], series-type double inverted pendulum systems [4,8,9], parallel-type double inverted pendulum systems [3,6,10], and so on. For a parallel-type double inverted pendulum system, stabilization control is impossible if the two pendulums have the same natural frequency. Since the natural frequency of a pendulum depends on the length of the pendulum, it is necessary for the two pendulums of a parallel-type double inverted

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0165-0114/02/$ - see front matter
PII: S0165-0114(01)00028-8

pendulum system to have different lengths. Furthermore, since the cart moving affects the two pendulums directly, the shorter pendulum with a higher natural frequency tends to respond intensively and is liable to fall down. Therefore, it is said [6] that stabilization control of a parallel-type double inverted pendulum system is the most difficult among the family.

Till now, several approaches have been studied for stabilization control of parallel-type double inverted pendulum system. Based on the singleton-type reasoning method and genetic algorithm, Fujita and Mizumoto [3] constructed a 4-input 1-output fuzzy controller only for balancing the two pendulums of a parallel-type double inverted pendulum system. Since the position control of the cart was not taken into consideration, a limitless rail was necessary to keep the two pendulums upright. Kawatani and Tamaguchi [6] linearized first the nonlinear mathematical model of a parallel-type double inverted pendulum system, and then designed a stabilization controller by a state feedback gain vector and full state observer. Although the controller worked well for small initial angles, stabilization was not guaranteed when the initial angles of the two pendulums slightly increased. Sugie and Okada [10] derived the linearized mathematical model of a circular parallel-type double inverted pendulum system, and created a stabilization controller based on the H^∞ loop shaping design procedure. Besides complicated mathematical analysis, however, stabilization results of the controller showed a lasting vibration with amplitude of about 3.0°.

In stabilization control of a parallel-type double inverted pendulum system, 6 input items are necessary in order to cover all of the angular controls of the two pendulums and the position control of the cart. Since the angular controls of the two pendulums should be done first before the position control of the cart from intuition, the priority orders of the three controls have to be discriminated clearly. The conventional fuzzy inference model which puts all of the input items into the antecedent part of each fuzzy rule, however, has difficulty to settle fuzzy rules of 6 input items and has poor ability to express the control priority orders. As a result, although many fuzzy controllers [2,5,7,11] have been proposed for stabilization of single inverted pendulum system, only few approaches based on the conventional fuzzy inference model are found for stabilization of series-type double inverted pendulum system [8] and parallel-type double inverted pendulum system [3].

On the other hand, in the SIRMs dynamically connected fuzzy inference model [12,14], a SIRM and a dynamic importance degree are defined for each input item. Since the input items can be processed dispersedly by the SIRMs and the control priority orders can be represented definitely by the dynamic importance degrees, the model has been successfully applied to trajectory tracking control [15] and stabilization control of single inverted pendulum systems [13].

In this paper, a new fuzzy controller for stabilizing a parallel-type double inverted pendulum system is presented based on the SIRMs dynamically connected fuzzy inference model. The fuzzy controller takes the normalized angles and angular velocities of the two pendulums, the normalized position and velocity of the cart as the input items, and takes the normalized driving force as the output item. Each input item is assigned with a SIRM and a dynamic importance degree. The SIRMs and the dynamic importance degrees are set up such that the angular control of the longer pendulum takes the highest priority over the angular control of the shorter pendulum and the position control of the cart when the angle of the longer pendulum is big. By using the SIRMs and the dynamic importance degrees, the priority orders of the three controls are automatically adjusted according to control situations. The fuzzy controller has a simple and intuitively understandable structure, and executes the three controls entirely in parallel. Simulation results show that the fuzzy controller can stabilize completely a parallel-type double inverted pendulum system for a wide range of the initial angles of the two pendulums in 10.0 s. This is the first result for a fuzzy controller to realize complete stabilization control of a parallel-type double inverted pendulum system.

2. Parallel-type double inverted pendulum system

As shown in Fig. 1, the parallel-type double inverted pendulum system considered here consists of a straight line rail, a cart moving on the rail, a longer pendulum 1 hinged on the right side of the cart, a shorter pendulum 2 hinged on the left side of the cart, and a driving unit. In the same vertical plane with the rail, the two pendulums can rotate freely around their own pivots.

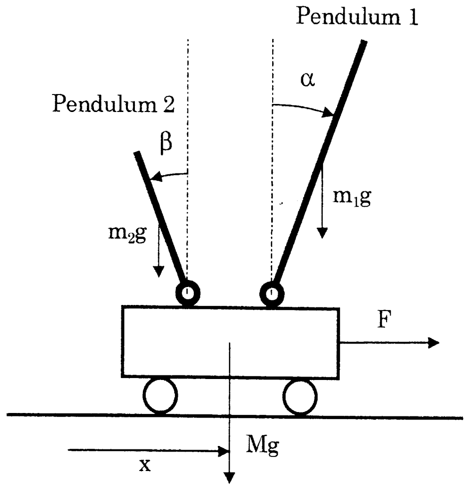

Fig. 1. Configuration of parallel-type double inverted pendulum system.

Here, the parameters M, m_1, m_2 (kg) are separately the masses of the cart, the longer pendulum 1 and the shorter pendulum 2, respectively. The parameter $g = 9.8\ \mathrm{m/s^2}$ is the gravity acceleration. Suppose the mass of each pendulum is distributed uniformly. The length from the gravity center of the longer pendulum 1 to its pivot is given as l_1 (m), which is equal to half the length of the longer pendulum 1. The length from the gravity center of the shorter pendulum 2 to its pivot is given as l_2 (m), which equals to half the length of the shorter pendulum 2.

The position of the cart from the rail origin is denoted as x, and is positive when the cart locates on the right side of the rail origin. The angles of the longer pendulum 1 and the shorter pendulum 2 from their upright positions are denoted separately as α and β, and clockwise direction is positive. The driving force applied horizontally to the cart is denoted as F (N), and right direction is positive. Also, suppose no friction exists in the pendulum system. Then the dynamic equation of such a parallel-type double inverted pendulum system can be obtained by Lagrange's equation of motion as

$$a_{11}\ddot{x} + a_{12}\ddot{\alpha} + a_{13}\ddot{\beta} = b_1,$$

$$a_{21}\ddot{x} + a_{22}\ddot{\alpha} = b_2,$$

$$a_{31}\ddot{x} + a_{33}\ddot{\beta} = b_3, \tag{1}$$

where the coefficients are given by

$$a_{11} = M + m_1 + m_2,$$

$$a_{12} = m_1 l_1 \cos\alpha,$$

$$a_{13} = m_2 l_2 \cos\beta,$$

$$a_{21} = a_{12},$$

$$a_{22} = 4m_1 l_1^2/3,$$

$$a_{31} = a_{13},$$

$$a_{33} = 4m_2 l_2^2/3, \tag{2}$$

$$b_1 = F + m_1 l_1 \dot{\alpha}^2 \sin\alpha + m_2 l_2 \dot{\beta}^2 \sin\beta,$$

$$b_2 = m_1 l_1 g \sin\alpha,$$

$$b_3 = m_2 l_2 g \sin\beta. \tag{3}$$

In the following control simulations, the state variables (the position x and velocity $\dot{x}$ of the cart, the angle α and angular velocity $\dot{\alpha}$ of the longer pendulum 1, the angle β and angular velocity $\dot{\beta}$ of the shorter pendulum 2) are calculated based on Euler's approximation method.

3. SIRMS dynamically connected fuzzy inference model

Before presenting the stabilization fuzzy controller, let us describe briefly the SIRMs dynamically connected fuzzy inference model [12,14] for systems of n input items and 1 output item.

As well known, the conventional fuzzy inference model, which puts all the input items into the antecedent part of each fuzzy rule, causes the total number of possible fuzzy rules to increase exponentially with the number of the input items and has difficulty in setting up each rule. To solve the problems, the SIRMs dynamically connected fuzzy inference model first defines a SIRM separately for each input item as

$$\text{SIRM-}i\text{:}\quad \{R_i^j\text{: if } x_i = A_i^j \text{ then } f_i = C_i^j\}_{j=\overline{1,m_i},\ i=\overline{1,n}}. \tag{4}$$

Here, SIRM-i denotes the SIRM of the ith input item, and R_i^j is the jth rule in the SIRM-i. The ith input item x_i is the only variable in the antecedent part, and the consequent variable f_i is an intermediate variable corresponding to the output item f. A_i^j and C_i^j are the membership functions of x_i and f_i in the jth rule of the SIRM-i. Further, $i=1,2,\ldots,n$ is the index number of the SIRMs, and $j=1,2,\ldots,m_i$ is the index number of the rules in the SIRM-i.

The inference result f_i^0 of the consequent variable f_i can be calculated based on the min–max-gravity method or product–sum-gravity method or simplified inference method. Since the consequent variables of the SIRMs all correspond directly to the output item, the simplest way to obtain the output value of the output item is just summing up the inference results of all the SIRMs. But this does not work well because each input item usually plays an unequal role in system performance. Among the input items, some may contribute significantly, while the contribution of the others may be relatively small. Some input items may improve system performance more if their roles are strengthened, while the others may not have a positive influence on system performance if emphasized.

To express clearly the different role of each input item in system performance, then, the SIRMs dynamically connected fuzzy inference model defines a dynamic importance degree w_i^{D} independently for each input item x_i $(i=1,2,\ldots,n)$ as

$$w_i^{\mathrm{D}} = w_i + B_i \Delta w_i^0. \tag{5}$$

On the right-hand side of Eq. (5), the first term and the second term separate the base value and the dynamic value. The base value w_i guarantees the minimum weight of the corresponding input item for a control process. The dynamic value, defined as the product of the breadth B_i and the inference result Δw_i^0 of the dynamic variable Δw_i, plays a role in tuning the degree of the influence of the input item on system performance according to control situation changes. The base value and the breadth are control parameters, and the dynamic variable can be described by fuzzy rules. Since the inference result of the dynamic variable is limited in $[0.0, 1.0]$, the dynamic importance degree will vary between $[w_i, w_i + B_i]$.

Suppose that each dynamic importance degree w_i^{D} and the fuzzy inference result f_i^0 of each SIRM are already calculated. Then, the SIRMs dynamically connected fuzzy inference model obtains the output value of the output item f by

$$f = \sum_{i=1}^{n} w_i^{\mathrm{D}} f_i^0 \tag{6}$$

as the summation of the products of the fuzzy inference result of each SIRM and its dynamic importance degree for all the input items.

As shown in Eq. (6), the model output is linear to the inference result of each SIRM. If the inference result of each SIRM is identical, then the contribution of one input item to the model output is controlled by its dynamic importance degree. Therefore, the input items with larger importance degrees will contribute more to the model output, while the input items with smaller importance degrees contribute less to the model output.

4. Stabilization fuzzy controller

Without losing generality, the rail origin is selected as the desired position of the cart. Then, the stabilization control of the parallel-type double inverted pendulum system is to balance the two pendulums upright and move the cart to the rail origin in short time.

The angle α and angular velocity $\dot{\alpha}$ of the longer pendulum 1, the angle β and angular velocity $\dot{\beta}$ of the shorter pendulum 2, the position x and velocity $\dot{x}$ of the cart normalized by their own scaling factors are selected in this order as the input items x_i $(i=1,2,\ldots,6)$. The driving force F normalized by its scaling factor is chosen as the output item f. If the six input items all converge to zero, then the stabilization control is apparently achieved. Here, a new fuzzy controller with the six input items and the output item for stabilizing the parallel-type double inverted pendulum system is constructed based on the SIRMs dynamically connected fuzzy inference model. By using the SIRMs, the total number of fuzzy rules can be reduced significantly and the fuzzy rules can be easily established. By using the dynamic importance degrees, the control priority orders of the two pendulums and the cart can be represented definitely. As a result, stabilizing the parallel-type double inverted pendulum system including the position control of the cart becomes possible.

4.1. Setting the SIRMs

As stated in Section 3, each input item is given with a SIRM and a dynamic importance degree in the SIRMs dynamically connected fuzzy inference model. The SIRMs of the input items in the stabilization control of the parallel-type double inverted pendulum system are considered here first.

In case of positive big values of the angle and angular velocity of the longer pendulum 1, if positive driving force is added to move the cart toward right direction, the longer pendulum 1 will rotate counterclockwise toward upright position. Due to its higher natural frequency, however, the shorter pendulum 2 will rotate counterclockwise quickly, causing the angle and angular velocity of the shorter pendulum 2 to become negative. If the angle of the shorter pendulum 2 is negative, the cart has to be moved toward left direction by negative driving force as will be discussed below. If negative driving force is applied, however, the longer pendulum 1 will change its rotation direction to clockwise and its angle and angular velocity will become positive again. As a result, the longer pendulum 1 will repeat rotating clockwise and counterclockwise. To stand up the longer pendulum 1 effectively in this case, therefore, it is necessary to move the cart toward left direction first by negative driving force. Although the longer pendulum 1 will then fall down clockwise further, the shorter pendulum 2 will also rotate clockwise faster enough for its angle and angular velocity to become positively larger than those of the longer pendulum 1. Then by moving the cart toward right direction, the two pendulums rotate counterclockwise with synchronization kept and are balanced both to their upright positions. Similarly, if the angle and angular velocity of the longer pendulum 1 are negative big, the cart has to be moved toward right position first by positive driving force. Therefore, the SIRMs of the two input items x_1 and x_2 corresponding to the angle and angular velocity of the longer pendulum 1 can be set up as in Table 1.

Table 1
SIRM setting for the longer pendulum 1

Antecedent variable x_i $(i=1,2)$	Consequent variable f_i $(i=1,2)$
NB	1.0
ZO	0.0
PB	−1.0

Table 2
SIRM setting for the shorter pendulum 2

Antecedent variable x_i $(i=3,4)$	Consequent variable f_i $(i=3,4)$
NB	−1.0
ZO	0.0
PB	1.0

When the angle and angular velocity of the shorter pendulum 2 are positive big, the shorter pendulum 2 will fall down clockwise increasingly at its angular velocity if no control action is done at once. If the cart is moved toward left direction, the shorter pendulum 2 will rotate acceleratingly clockwise so that the angle of the shorter pendulum 2 gets even bigger and standing up the shorter pendulum 2 becomes impossible. To balance the shorter pendulum 2 upright in this case, therefore, it is necessary to move the cart toward right direction by positive driving force so as to rotate the shorter pendulum 2 counterclockwise. In the same way, if the angle and angular velocity of the shorter pendulum 2 are negative big, the cart has to be moved toward left direction by negative driving force. Since the longer pendulum 1 has a lower natural frequency, it will rotate at an angular velocity basically smaller than that of the shorter pendulum 2 during this period. The shorter pendulum 2 rotates faster because of its higher natural frequency and becomes synchronized with the longer pendulum 1. Then the two pendulums rotate toward the same direction and get balanced upright. Therefore, the SIRMs of the two input items x_3 and x_4 corresponding to the angle and angular velocity of the shorter pendulum 2 are established as shown in Table 2.

Suppose the two pendulums are already balanced upright. In case of positive values of the position and velocity of the cart, if the cart is moved toward right direction by positive driving force, the shorter pendulum 2 will rotate counterclockwise faster because of its higher natural frequency. Consequently, the angle and angular velocity of the shorter pendulum 2 become negative and are bigger than those of the longer pendulum 1 in magnitude. For negative values of the angle and angular velocity of the shorter pendulum 2,

Table 3
SIRM setting for the cart

Antecedent variable x_i $(i=5,6)$	Consequent variable f_i $(i=5,6)$
NB	1.0
ZO	0.0
PB	−1.0

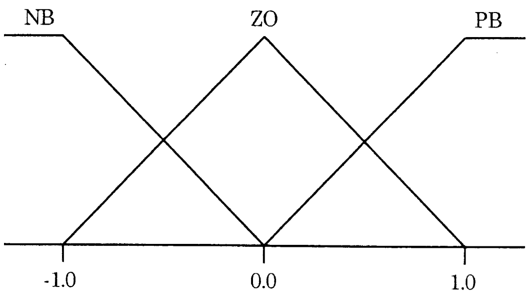

Fig. 2. Membership functions for each SIRM.

the two SIRMs of Table 2 generate negative driving force to move the cart toward left direction so that the two pendulums are rotated clockwise. When the angle and the angular velocity of the shorter pendulum 2 become positively bigger than those of the long pendulum 1, the two SIRMs of Table 2 output positive driving force to move the cart toward right direction. As a result, the cart moves right and left alternately, and is likely to move outside gradually. Therefore, if the position and velocity of the cart are positive, negative driving force should first be added to the cart. After the angle and angular velocity of the short pendulum 2 become positive, positive driving force from the two SIRMs of Table 2 makes the shorter pendulum 2 rotate counterclockwise. When the angle and angular velocity of the short pendulum 2 turn negative, negative driving force from the two SIRMs of Table 2 balances the two pendulums toward their upright positions and puts the cart back to the rail origin. On the contrary, if the position and velocity of the cart are negative, positive driving force should be applied first to the cart. Therefore, the SIRMs of the two input items x_5 and x_6 corresponding to the position and velocity of the cart can be given in Table 3.

Here, the membership functions NB, ZO, PB of each antecedent variable are defined in Fig. 2 as triangle or trapezoids. The consequent variables f_i $(i=1,2,\dots,6)$ are intermediate variables, all corresponding to the output item f of the fuzzy controller. Since the simplified reasoning method is adopted here, real numbers are assigned as singleton-type membership functions to the consequent variable of each SIRM.

4.2. Control priorities and the dynamic importance degrees

As it is shown in Tables 1–3, all SIRMs infer the output item of the fuzzy controller. The angular control of the shorter pendulum 2 by Table 2 rotates the shorter pendulum 2 directly toward upright position. However, the angular control of the longer pendulum 1 by Table 1 makes the longer pendulum 1 fall down further and the position control of the cart by Table 3 moves the cart away from the rail origin. By this setting of Tables 1 and 3, the shorter pendulum 2 as a result inclines to the same side with the longer pendulum 1 and has a larger angle because of its higher natural frequency. As the shorter pendulum 2 is balanced upright by using Table 2 then, the longer pendulum 1 is also balanced upright and the cart is moved to the rail origin. From this point of view, the angular control of the longer pendulum 1 and the position control of the cart are realized indirectly.

To make the indirect control of the longer pendulum 1 and the cart is also feasible, the angular control of the longer pendulum 1 and the position control of the cart should be discriminated from the angular control of the shorter pendulum 2. As well known, the longer pendulum 1 has a bigger momentum, while the shorter pendulum 2 has a higher natural frequency. When the angle of the longer pendulum 1 is big, the longer pendulum 1 will fall down further and make it more difficult to balance the long pendulum 1 upright because of its bigger momentum if the angular control of the longer pendulum 1 is not done immediately. When the angle of the shorter pendulum 2 is big, because of its higher response characteristic, it is relatively easy to stand up the shorter pendulum 2 again if relevant control action is executed. Therefore, if the angle of the longer pendulum 1 is big, the angular control of the longer pendulum 1 should be done first with the highest priority so that the shorter pendulum 2 inclines to the same side with the longer pendulum 1. If the angle of the shorter pendulum 2 is big and the two pendulums are located on different sides, then the angular control of the shorter pendulum 2 should be executed so that the two pendulums get inclined to the same side. By performing the angular control of the shorter pendulum 2 then, the two pendulums are balanced upright. After the two pendulums are almost stood up, the position control of the cart can be started.

In the stabilization control of the parallel-type double inverted pendulum system, therefore, the angular control of the longer pendulum 1 should take the highest priority when its angle is big. When the angle of the shorter pendulum 2 becomes big, the angular control of the shorter pendulum 2 should have the highest priority. The position control of the cart should have the lowest priority before the two pendulums are balanced upright. To make the stabilization control effective, the priority orders of the angular control of the longer pendulum 1, the angular control of the shorter pendulum 2, and the position control of the cart should be reflected in calculation of the output value of the output item. Equal control priorities will apparently cause contradictions among the angular control of the longer pendulum 1, the angular control of the shorter pendulum 2, and the position control of the cart.

On the other hand, the dynamic importance degrees indicate the influence strengths of the input items on system performance and can then express definitely the priority orders. The bigger the value of a dynamic importance degree is, the higher the priority order of the corresponding input item becomes. In the stabilization control, the longer pendulum 1, the shorter pendulum 2, and the cart have two input items each. Therefore, the control priority orders of the angular control of the longer pendulum 1, the angular control of the shorter pendulum 2, and the position control of the cart are represented by the two dynamic importance degrees of their own two input items.

4.3. Setting the dynamic variables of the dynamic importance degrees

As defined in Eq. (5), each dynamic importance degree has two control parameters and one dynamic variable. In this section, the fuzzy rules for the dynamic variables of the six dynamic importance degrees are discussed first.

Table 4
Fuzzy rules for the two dynamic variables of the longer pendulum 1

Antecedent variable $\lvert x_1 \rvert$	Consequent variable $\Delta w_1, \Delta w_2$
DS	0.0
DM	0.5
DB	1.0

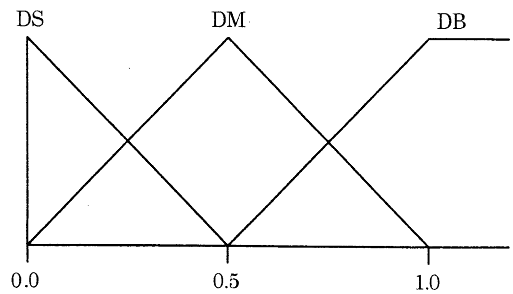

Fig. 3. Membership functions for each dynamic variable.

For the longer pendulum 1, the fuzzy rules for the dynamic variables Δw_1 and Δw_2 of the dynamic importance degrees w_1^D and w_2^D of the input items x_1 and x_2 can be established as in Table 4 by selecting the absolute value of the input item x_1 as the only antecedent variable. Here, the membership functions DS, DM, DB are defined in Fig. 3. By this setting, when the absolute value of the input item x_1 corresponding to the angle of the longer pendulum 1 is big, the inference results of the two dynamic variables will both become big. Therefore, the values of the two dynamic importance degrees increase so much that the angular control of the longer pendulum 1 is emphasized. If the absolute value of the angle of the longer pendulum 1 is near zero, the inference results of the two dynamic variables will both become almost zero, and the two dynamic importance degrees will approach their base values. As a result, the influence strength of the angular control of the longer pendulum 1 is weakened.

Similarly for the shorter pendulum 2, the fuzzy rules for the dynamic variables Δw_3 and Δw_4 of the dynamic importance degrees w_3^D and w_4^D of the input items x_3 and x_4 can be set as in Table 5 by selecting the absolute value of the input item x_3 as the sole antecedent variable. When the absolute value of the

Table 5
Fuzzy rules for the two dynamic variables of the shorter pendulum 2

Antecedent variable $\|x_3\|$	Consequent variable $\Delta w_3, \Delta w_4$
DS	0.0
DM	0.5
DB	1.0

Table 6
Fuzzy rules for the two dynamic variables of the cart

$\Delta w_5, \Delta w_6$		$\|x_1\|$		
		DS	DM	DB
$\|x_3\|$	DS	1.0	0.5	0.0
	DM	0.5	0.0	0.0
	DB	0.0	0.0	0.0

input item x_3 corresponding to the angle of the shorter pendulum 2 is big, the inference results of the two dynamic variables will also get big so that the two dynamic importance degrees of the shorter pendulum 2 increase much. When the absolute value of the angle of the shorter pendulum 2 is small, the inference results of the two dynamic variables will also get small so that the two dynamic importance degrees of the shorter pendulum 2 decrease much. Consequently, the control priority order of the shorter pendulum 2 is adjusted according to the situation of its angle.

If the position control of the cart is started before the two pendulums are not stood up yet, the state of the two pendulums may be destroyed. Therefore, the dynamic variables Δw_5 and Δw_6 of the dynamic importance degrees w_5^{D} and w_6^{D} of the input items x_5 and x_6 corresponding to the position and velocity of the cart can be described by the fuzzy rules in Table 6. Here, the absolute values of the input items x_1 and x_3 are used as the antecedent variables. In Table 6, the real number output of the consequent part is set up to 0.0 in those fuzzy rules of $|x_1| = \mathrm{DB}$ or $|x_3| = \mathrm{DB}$. By this setting, when the two pendulums are almost stood up, both inference results of the two dynamic variables will become big. As a result, the values of the two dynamic importance degrees of the cart will increase relatively, making the position control of the cart become possible. If one of the two pendulums is still not balanced upright, both the inference results of the dynamic variables will be small. As a result, the values of the two dynamic importance degrees of the cart decrease so that the position control of the cart has low priority order.

4.4. Setting the control parameters of the dynamic importance degrees

Since the SIRMs and the dynamic variables of the dynamic importance degrees all have been set up, the structure of the proposed fuzzy controller becomes clear. As stated above, however, each dynamic importance degree also has two control parameters, i.e., the base value and the breadth. The rule setting of the dynamic variables only does not guarantee the necessary control priority orders. The control parameters also have to be adequately set up.

Since the natural frequencies of the two pendulums have a strong influence on the stabilization control performance, the pendulum system with different pendulum lengths should have different set of the control parameters. Since proposing the structure of the fuzzy controller for the stabilization control of the parallel-type double inverted pendulum system is the principal objective of this paper, how to set up the control parameters in a systematic manner will be a future subject.

However, some relations among the control parameters of the dynamic importance degrees can be obtained from the control priority orders. For the sake of the longer pendulum 1 to take the highest control priority order when its angle is big, the sum of the base value and the breadth of each dynamic importance degree of the longer pendulum 1 should be larger than that of the other dynamic importance degrees. For the sake of the shorter pendulum 2 to take the highest control priority order when its angle is big, the sum of the base value and the breadth of each dynamic importance degree of the shorter pendulum 2 should be larger than that of the dynamic importance degrees of the cart. To start the position control of the cart without disturbing the upright state of the two pendulums, the sum of the base value and the breadth of either dynamic importance degree of the cart should be rather smaller.

Table 7
Control parameters of the dynamic importance degrees

Input item	Base value	Breadth
x_1	2.3694	0.5278
x_2	3.5874	0.0578
x_3	1.9398	0.5298
x_4	1.4012	1.2148
x_5	0.3281	0.0000
x_6	0.0328	0.2690

Here, the mass and half-length of the longer pendulum 1, the mass and half-length of the shorter pendulum 2, and the car mass are selected separately as $m_1 = 0.3$ kg, $l_1 = 0.6$ m, $m_2 = 0.1$ kg, $l_2 = 0.2$ m, and $M = 1.0$ kg. The scaling factors of the input items are set up to 15.0°, 100.0°/s, 15.0°, 100.0°/s, 2.4 m, 1.0 m/s, respectively. The scaling factor of the output item is defined as 10.0 times the total mass of the two pendulums and the cart. Compared with the single inverted pendulum systems [13], the scaling factors of the two angles are reduced to half while the others keep unchanged just because the controllable ranges of the two angles are narrower.

To tune automatically the control parameters, the random optimization search method [1] is adopted. In each step, the sampling period and total control time are separately fixed to 0.01 and 25.0 s. The initial angle of the longer pendulum 1 is set up to 5.0°, while the initial values of the other state variables are set up to zeros. All of the base values and the breadths of the dynamic importance degrees are initially set up to zeros. The random optimization search is run for 40,000 steps along such direction, that the total summation of the absolute values of all state variables and the driving force at each sampling time from the beginning to the end of the total control time is reduced. The base values and the breadths after the random optimization search are shown in Table 7.

As it can be seen from Table 7, the sum of the base value and the breadth of either input item of the longer pendulum 1 is larger than that of any other input items. The sum of the base value and the breadth of either input item of the shorter pendulum 2 is much larger than that of either input item of the cart. Apparently, the control parameters reflect the necessary control priority orders very well.

4.5. Features of the fuzzy controller

Fig. 4 shows the block diagram of the fuzzy control system for the stabilization control of the parallel-type double inverted pendulum system. The state variables $\alpha, \dot{\alpha}, \beta, \dot{\beta}, x, \dot{x}$ from the pendulum system are fed back and compared with the reference inputs. Since the desired values of the reference inputs are all zeros in the stabilization control, the variables are reversely inputted into the normalizer block. The normalizer block normalizes the state variables by their scaling factors and creates the input items x_i $(i = 1, 2, \ldots, 6)$. Each input item x_i is then fed to the SIRM-i block, where the fuzzy inference of the SIRM corresponding to the input item x_i is done. The two dynamic importance degree (DID) blocks of the longer pendulum 1 take the absolute value of the input item x_1 as their antecedent variable, and the two DID blocks of the shorter pendulum 2 take the absolute value of the input item x_3 as their antecedent variable. The two DID blocks of the cart use both the absolute values of the input items x_1 and x_3 as their antecedent variables. The DID-i block calculates the value of the dynamic importance degree of the input item x_i. After the output of each SIRM-i block is multiplied by the output of the DID-i block, summing them for all the input items gives the output value of the output item f of the fuzzy controller. The output scaling factor (OSF) block finally multiplies the output value of the output item of the fuzzy controller by its scaling factor to generate the actual driving force F to the cart.

Although there are six blocks for the SIRMs and six blocks for the dynamic importance degrees in the fuzzy controller, each block performs simple processing only. Further, each SIRM block infers an intermediate variable directly related with the output item of the fuzzy controller, and each DID block adjusts the value of its dynamic importance degree according to control situations. The output of each SIRM block multiplied by the output of the corresponding DID block is actually a part of the output value of the output item of the fuzzy controller. Therefore, the angular control of the longer pendulum 1, the angular control of the shorter pendulum 2, and the position control of the cart are performed completely in parallel.

If the absolute value of the angle of the longer pendulum 1 is bigger, the angular control of the longer pendulum 1 will take the highest priority because the

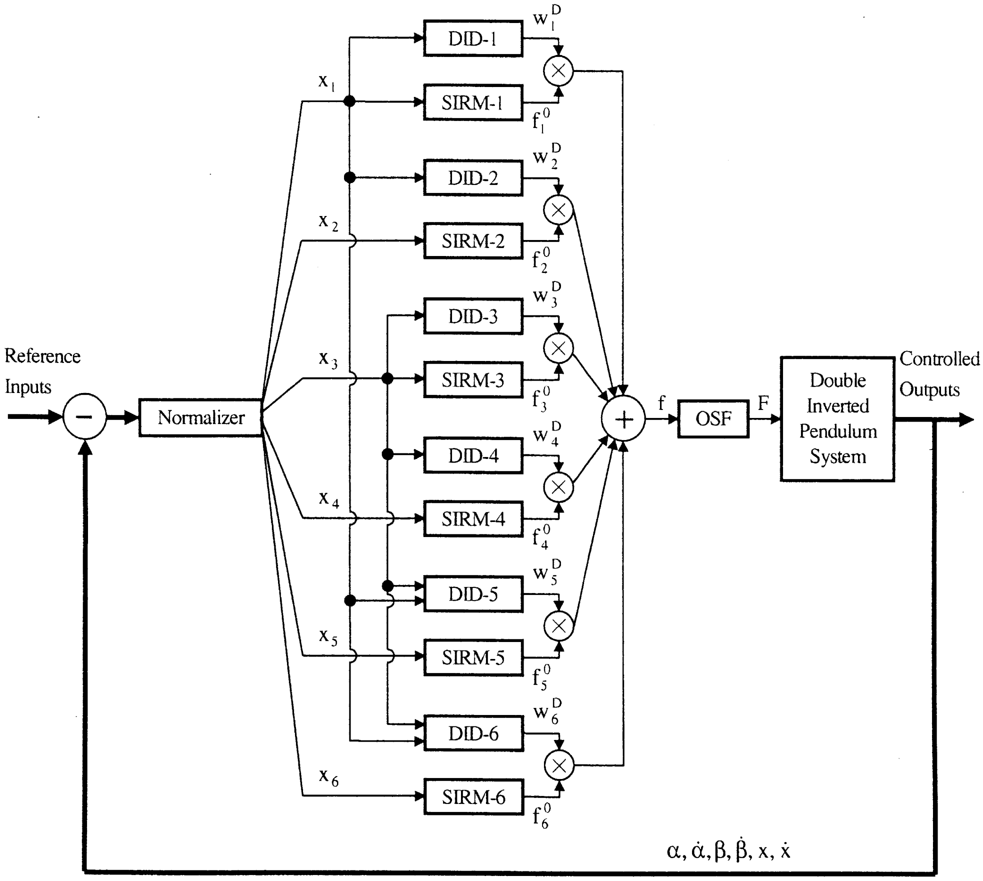

Fig. 4. Block diagram of the stabilization fuzzy control system.

two dynamic importance degrees of the longer pendulum 1 become the biggest. If the absolute value of the angle of the longer pendulum 1 is smaller and the absolute value of the angle of the shorter pendulum 2 is bigger, the two dynamic importance degrees of the shorter pendulum 2 increase and the inference result of the SIRM corresponding to the angle of the shorter pendulum 2 becomes larger. Although the two dynamic importance degrees of the longer pendulum 1 are still larger because of its larger base values, the inference result of the SIRM corresponding to the angle of the longer pendulum 1 becomes smaller. As a result, the contribution of the input items x_3 and x_4 in Eq. (6) will exceed that of the input items x_1 and x_2 so that the angular control of the shorter pendulum 2 becomes the main. If both the absolute values of the angles of the two pendulums are small, the inference results of the two SIRMs corresponding to the two angles will become small. At the same time, the two dynamic importance degrees of the cart increase, while the dynamic importance degrees of the two pendulums decrease. Consequently, the contribution of the input items x_5 and x_6 in Eq. (6) will increase relatively, making it possible to start the position control of the cart. By using the SIRMs and adjusting the value of each dynamic importance degree according to control situations, therefore, smooth switching among the three controls is realized automatically and makes the stabilization control effective.

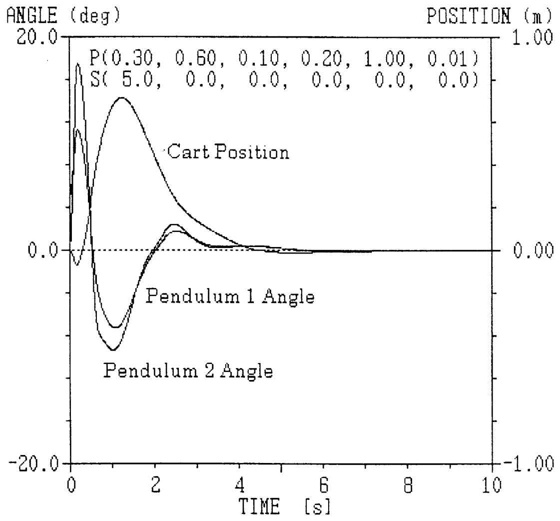

Fig. 5. Control result for initial angles 5.0° and 0.0°.

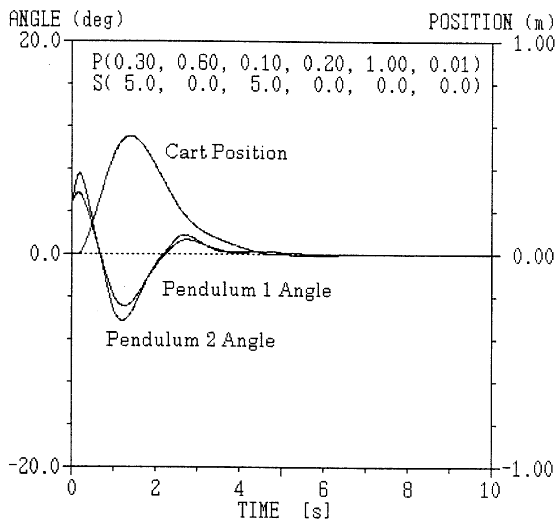

Fig. 6. Control result for initial angles 5.0° and 5.0°.

5. Stabilization control simulations

To verify the effectiveness of the proposed stabilization fuzzy controller, several control simulations are done. Figs. 5, 6, 7 show the control results in order when the initial angles α and β of the two pendulums are separately set up to 5.0° and 0.0°, 5.0° and 5.0°, 5.0°, and 10.0°, while the initial values of the other state variables are all fixed to zeros. By the way,

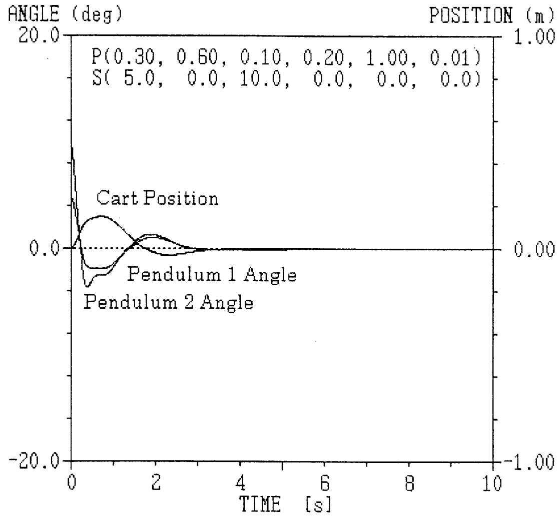

Fig. 7. Control result for initial angles 5.0° and 10.0°.

the initial state in Fig. 5 corresponds to that used in the random optimization search. The left axis and the right axis separately represent the angle of the pendulums and the position of the cart. The values in $P(0.30, 0.60, 0.10, 0.20, 1.00, 0.01)$ in this order, stand for, the mass and half-length of the longer pendulum 1, the mass and half-length of the shorter pendulum 2, the cart mass, and the sampling period, respectively. The values in $S(5.0, 0.0, 10.0, 0.0, 0.0, 0.0)$ denote the initial values of the angle and angular velocity of the longer pendulum 1, the angle and angular velocity of the shorter pendulum 2, the position and velocity of the cart, respectively.

In Fig. 5, the angular control of the longer pendulum 1 takes the priority over the other two controls at control action beginning because the initial angle of the longer pendulum 1 is bigger than the initial angle of the shorter pendulum 2. Negative driving force is first generated based on the SIRM of the angle of the longer pendulum 1, and moves the cart left. Although the longer pendulum 1 resultantly falls down from 5.0° to 11.0° further, the shorter pendulum 2 becomes inclined to about 17.0°. Since the contribution of the shorter pendulum 2 exceeds that of the longer pendulum 1 in the driving force from then, the angular control of the shorter pendulum 2 becomes the main. Consequently, the cart is moved right by positive driving force of a maximum of about 25.0 N, so that the two pendulums get synchronized to rotate toward the same direction. Finally, the two pendulums are gradually balanced to their upright positions and the cart is returned to the rail origin. If the time interval from control action beginning to such a condition that all the state variables just converge to 0.1°, 0.1°/s, 0.1°, 0.1°/s, 0.01 m, 0.01 m/s each is defined as complete stabilization time, the complete stabilization time of Fig. 5 is about 6.23 s.

In Fig. 6, although the initial angles of the two pendulums are the same, the angular control of the longer pendulum 1 is done first at control action beginning because the two dynamic importance degrees of the longer pendulum 1 are larger. Since negative driving force is generated for positive angle of the longer pendulum 1, the cart is moved a little left (although the moving distance is too short to read from Fig. 6). Consequently, the angle of the longer pendulum 1 increases to about 6.0°, and the angle of the shorter pendulum 2 increases to about 8.0°. From then, the angular control of the shorter pendulum 2 comes to take the highest priority because the shorter pendulum 2 contributes more to the driving force than the longer pendulum 1. Therefore, positive driving force is applied to move the cart right, and the two pendulums begin to rotate with synchronization. By switching the three controls with control situations smoothly, the pendulum system is finally completely stabilized in about 6.12 s. The maximum driving force is only about 3.0 N in this case.

As the initial state in Fig. 7, the two pendulums incline to the same side and the initial angle of the shorter pendulum 2 is twice as large as that of the longer pendulum 1. According to the SIRM setting of the two angles, the inference result of the SIRM corresponding to the initial angle of the shorter pendulum 2 also becomes twice as large as that corresponding to the initial angle of the longer pendulum 1. From Tables 4, 5 and 7, the value of the dynamic importance degree corresponding to the initial angle of the shorter pendulum 2 is almost equal to that corresponding to the initial angle of the longer pendulum 1. Therefore, at control action beginning at the angular control of the shorter pendulum 2 is strengthened first so that the cart is moved right by positive driving force. As a result, the two pendulums keep synchronized from control action beginning and converge gradually to their upright positions. In this case, the complete stabilization time is about 5.29 s and the maximum driving force is less than 10.0 N.

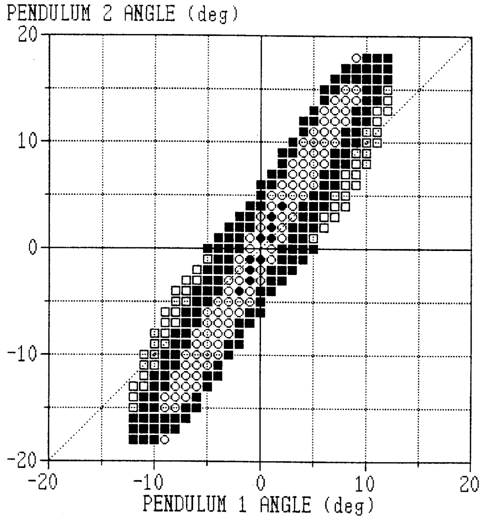

Fig. 8. Stabilization domain of the two pendulums 1.2 and 0.4 m.

Fig. 8 shows the stabilization domain of the initial angles of the two pendulums, for which the proposed fuzzy controller can stabilize the pendulum system. Here, the horizontal axis and the vertical axis stand separately for the initial angles of the longer pendulum 1 and the shorter pendulum 2. The initial angles of the two pendulums both are selected every 1.0° from −20.0° to +20.0°, and the initial values of the other state variables are all fixed to zeros. The symbols (●), (◯), (■), (□) mean that the complete stabilization time is within 4.0, or 6.0, or 8.0, or 10.0 s, respectively. Further, the failure limits of the angles of the two pendulums and the position of the cart are set up to [−20.0°, +20.0°], [−20.0°, +20.0°], [−2.4 m, +2.4 m], respectively. If any of the failure limits is broken off during a simulation, then it is regarded as a failure.

As it can be seen from Fig. 8, for all sets of the initial angles of the two pendulums in the stabilization domain, the proposed fuzzy controller can completely stabilize the pendulum system in 10.0 s. For several sets of the initial angles, the fuzzy controller can even stabilize the pendulum system in 4.0 s. For the longer pendulum 1 with an initial angle between [−5.0°, +5.0°], if the shorter pendulum 2 initially inclines further for up to 5.0° outside the longer pendulum 1, the pendulum system can be stabilized completely in 6.0 s. For the longer pendulum 1 with an initial angle between [−10.0°, +10.0°], if the initial angle of the shorter pendulum 2 is selected in a range of totally about 10.0° around the initial angle of the longer pendulum 1, the stabilization control of the pendulum system is possible.

It is also found that the stabilization domain in Fig. 8 as a whole leans a little upon the 45° line, i.e., toward the vertical axis of the angle of the shorter pendulum 2. The domain under the 45° line means that the initial angle of the longer pendulum 1 is larger than the initial angle of the shorter pendulum 2. For a set of the initial angles in the domain under the 45° line, the proposed fuzzy controller will first do the angular control of the longer pendulum 1. Resultantly, the shorter pendulum 2 gets synchronized with the longer pendulum 1 and the angle of the shorter pendulum 2 becomes bigger than that of the longer pendulum 1. In this case, since the driving force becomes larger with the increase in the difference between the initial angles of the two pendulums, the shorter pendulum 2 will swing intensively and fall down because of its higher response characteristic. On the other hand, the domain upon the 45° line means that the initial angle of the shorter pendulum 2 is larger than the initial angle of the longer pendulum 1. For a set of the initial angles in the domain upon the 45° line, if the initial angle of the shorter pendulum 2 is rather near that of the longer pendulum 1, the fuzzy controller will still give the highest priority to the angle control of the longer pendulum 1. Since the driving force is small in this case, however, the angle of the shorter pendulum 2 will not increase much. If the initial angle of the shorter pendulum 2 is rather larger than that of the longer pendulum 1, the fuzzy controller will first do the angular control of the shorter pendulum 2 directly at control action beginning. Consequently, the shorter pendulum 2 will rotate toward its upright position rather than rotating outside, and the two pendulums are kept synchronized from control action beginning. In this case, the stabilization control is relatively easy to be realized. Therefore, the stabilization domain concentrates mainly in the part upon the 45° line.

In the above simulations, the length ratio of the longer pendulum 1 to the shorter pendulum 2 is 3.0. In fact, control simulations are also done for the parallel-type double inverted pendulum system with different lengths of the two pendulums. It is found that if the length ratio of the longer pendulum 1 to the shorter pendulum 2 is between 2.0 and 5.0, the control parameters can be obtained through the random optimization search and the pendulum system can be completely stabilized by the proposed fuzzy controller. If the length ratio of the two pendulums is smaller than 2.0, the stabilization control becomes difficult because the difference in the natural frequencies of the two pendulums is too small for the two pendulums to get synchronized. On the other hand, if the length ratio of the two pendulum is larger than 5.0, the natural frequency of the shorter pendulum 2 is much larger than that of the longer pendulum 1. In this case, stabilizing the pendulum system is also difficult because the shorter pendulum 2 responds intensively even if the cart moves a little.

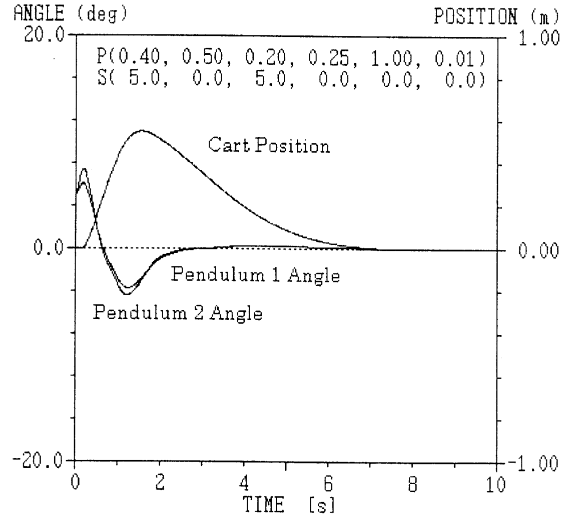

Fig. 9. Control example of the two pendulums 1.0 and 0.5 m.

Fig. 9 indicates a control example of the length ratio 2.0, where the two pendulums are separately 1.0 and 0.5 m long. The initial angles of the two pendulums are both set up to 5.0°. The two pendulums are balanced smoothly, and the pendulum system is completely stabilized in 7.24 s. Fig. 10 depicts a control example of the length ratio 5.0, where the lengths of the two pendulums are 2.0 and 0.4 m. The initial angles of the two pendulums are set up to 3.0° and −3.0°, respectively. This is a difficult situation. The angle of the shorter pendulum 2 increases from −3.0° to nearly 15.0° from control action beginning and then becomes

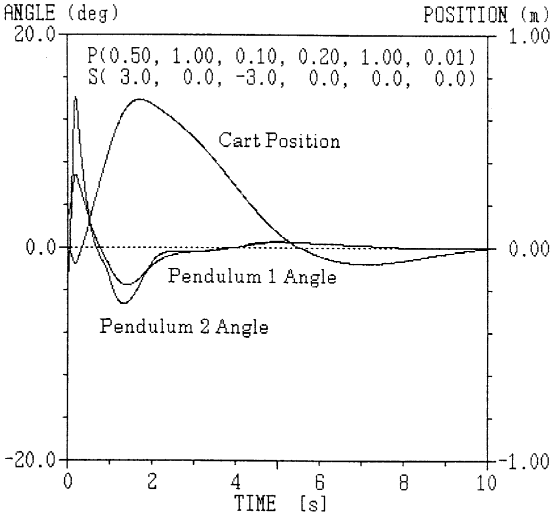

Fig. 10. Control example of the two pendulums 2.0 and 0.4 m.

synchronized with the longer pendulum 1. It takes 10.55 s to stabilize the pendulum system completely.

6. Conclusions

A stabilization fuzzy controller for the parallel-type double inverted pendulum system is proposed based on the SIRMs dynamically connected fuzzy inference model. The fuzzy controller takes the angle and angular velocity of the longer pendulum 1, the angle and angular velocity of the shorter pendulum 2, and the position and velocity of the cart as the input items, and takes the driving force as the output item. Each input item is assigned with a SIRM and a dynamic importance degree. The SIRMs and the dynamic importance degrees are designed such that the angular control of the longer pendulum takes the highest priority over the angular control of the shorter pendulum and the position control of the cart when the angle of the longer pendulum is big. By using the SIRMs and the dynamic importance degrees, the priority orders of the three controls are automatically adjusted according to control situations and the three controls are executed entirely in parallel. Simulation results show that the fuzzy controller completely stabilizes the parallel-type double inverted pendulum system in 10.0 s.

Since the conventional fuzzy inference model has difficulty to set up all fuzzy rules of 6 input items and expresses clearly the difference of the input items, the complete stabilization control of the parallel-type double inverted pendulum system based on the conventional fuzzy inference model is not yet found. It is verified in this paper, however, that based on the SIRMs dynamically connected fuzzy inference model, the proposed fuzzy controller has a simple and intuitively understandable structure, and can stabilize completely a parallel-type double inverted pendulum system in relatively short time. This is the first result for a fuzzy controller to achieve complete stabilization control of the parallel-type double inverted pendulum system.

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