A proposal of SIRMs dynamically connected fuzzy inference model for plural input fuzzy control

Jianqiang Yi\textsuperscript{a, *}, Naoyoshi Yubazaki\textsuperscript{b}, Kaoru Hirota\textsuperscript{c}

\textsuperscript{a}Laboratory of Complex Systems and Intelligent Science, Institute of Automation, Chinese Academy of Sciences, Beijing 100080, People’s Republic of China

\textsuperscript{b}Technology Research Center, Mycom, Inc., 12, S. Shimobano-cho, Saga Hirotsawa, Ukyo-ku, Kyoto 616-8303, Japan

\textsuperscript{c}Interdisciplinary Graduate School of Science and Engineering, Tokyo Institute of Technology 4259 Nagatsuta-cho, Midori-ku, Yokohama 226-8502, Japan

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Abstract

Single input rule modules (SIRMs) dynamically connected fuzzy inference model is proposed for plural input fuzzy control. For each input item, a SIRM is constructed and a dynamic importance degree is defined. The dynamic importance degree consists of a base value insuring the role of the input item through a control process, and a dynamic value changing with control situations to adjust the dynamic importance degree. Each dynamic value can be easily tuned based on the local information of current state. The model output is obtained by summarizing the products of the dynamic importance degree and the fuzzy inference result of each SIRM. The controller constructing method for constant value control systems is given, and constant value controls of typical first- and second-order lag plants are tested. The simulation results show that by using the proposed mode, the reaching time can be reduced by more than 15\% without any steady-state error, overshoot, or vibration compared with the SIRMs fixed importance degree connected fuzzy inference model. The proposed model is further successfully applied to stabilization control of an inverted pendulum system including the position control of the cart. © 2002 Elsevier Science B.V. All rights reserved.

Keywords: Dynamic importance degree; Fuzzy control; Fuzzy inference model; Pendulum; Process control; SIRM

1. Introduction

In the conventional IF–THEN fuzzy inference model [9], all the input items of systems are usually put into the antecedent part of each fuzzy rule. Consequently, both the numbers of fuzzy rules and parameters increase exponentially, and defining fuzzy rules becomes extremely difficult for large-scale systems. To solve the problems, the single input rule modules (SIRMs) fixed importance degree connected fuzzy inference model (the SIRMs fixed model for short) [18,19] was proposed, which constructed a SIRM and defined a fixed importance degree for each input item. When applied to function identification, the SIRMs fixed model sharply reduced the number of both the fuzzy rules and parameters. When applied to control systems such as first- and second-order lag plants, it also achieved some improvements in
control performance compared with the conventional fuzzy inference model. Since its importance degrees reflecting the influence of the input items on system performance were fixed through the whole control process including the rise stage and the settling stage, it was difficult to improve the control performance further. To obtain even better performance, the importance degrees should be dynamically tuned with continuously changing control situations.

Several researches have been done on dynamic adjustment of fuzzy controllers. The multi-unit controller [8] divided state space into several parts, each part is assigned with a fuzzy rule module. The time-variant fuzzy control method [12] changed the weight of singleton-type output in the consequent part or switches fuzzy rule modules from one to another according to time schedule. The dynamic fuzzy control method [17] prepared several fuzzy rule modules for different control situations and then used a time-series concept to unite all the modules. For each of the above methods, however, all the modules must be constructed and the system performance through the whole control process must be well understood in advance.

Furthermore, Chen [1] described an adaptive fuzzy controller to simplify the membership function constructions and the rule developments by genetic algorithm [3]. Godjevac [2] presented an adaptive fuzzy controller for robot navigation, in which the membership functions are tuned by a supervised learning procedure. Margaliot [10] designed an adaptive fuzzy controller via fuzzy Lyapunov synthesis, where the fuzzy rules are adapted based on the error between the desired and actual behavior of the Lyapunov function. These approaches determined automatically the parameters of the membership functions or the fuzzy rules; however, all the parameters were fixed after adaptation. To stabilize an inverted pendulum system [4–6], for example, the pendulum angular control should have priority over the cart position control when the pendulum is not balanced yet, and the control priority orders should be changed with control situations. Because the conventional fuzzy inference model was adopted and the input items were all treated equally, the adapted fuzzy controllers were difficult to deal with such a control problem.

In this paper, SIRMs dynamically connected fuzzy inference model is newly proposed for plural input fuzzy control. For each input item, a dynamic importance degree is given and a SIRM is constructed. The dynamic importance degree is defined as the sum of a base value insuring the role of the input item through a control process, and a dynamic value varying with control situations to control the influence of the input item. The dynamic value can be tuned in real-time style just based on the local information of current state. The SIRMs process the input items dispersedly and the dynamic importance degrees express the control priority orders definitely. The model output is obtained by summarizing the products of the dynamic importance degree and the fuzzy inference result of each SIRM. The building method of the fuzzy controllers based on the proposed model is given for constant value control systems, and control simulations of first- and second-order lag plants are done. The simulation results show that compared with the SIRMs fixed model, by using the proposed model the reaching time can be reduced by more than 15% and the value of the integral of time multiplied by absolute error (ITAE) can be decreased by more than 20%. The proposed model is further applied to the stabilization control of an inverted pendulum system including the position control of the cart. By using the SIRMs and the dynamic importance degrees, the proposed model is shown to completely stabilize the inverted pendulum system in short time by smoothly switching the priority orders of the pendulum angular control and the cart position control.

In Section 2, the SIRMs dynamically connected fuzzy inference model is presented in detail. In Section 3, the building method of the fuzzy controller for constant value control is indicated. In Section 4, the proposed model is applied to first- and second-order lag plants. In Section 5, stabilization control of an inverted pendulum system is done based on the proposed model.

2. SIRMs dynamically connected fuzzy inference model

A system with \( n \) input items and 1 output item is considered here for convenience. However, this can be easily extended to systems with plural output items.
Since there are \( n \) input items, totally \( n \) SIRMs are constructed first as

SIRM-1: \( \{ R_i^1: \text{if } x_i = A_i^1 \text{ then } \Delta u_i = C_i^1 \} \) for \( j = 1, \ldots \),

\( \ldots \)

SIRM-\( i \): \( \{ R_i^i: \text{if } x_i = A_i^i \text{ then } \Delta u_i = C_i^i \} \) for \( j = 1, \ldots \),

\( \ldots \)

SIRM-\( n \): \( \{ R_i^n: \text{if } x_n = A_n^i \text{ then } \Delta u_n = C_n^i \} \) for \( j = 1, \ldots \).

(1)

Each SIRM corresponds separately to one of the \( n \) input items. Here, SIRM-\( i \) means the SIRM referred to the \( i \)th input item, and \( R_i^j \) is the \( j \)th rule in the SIRM-\( i \). \( x_i \) corresponding to the \( i \)th input item is the variable in the antecedent part, and \( \Delta u_i \) is the variable in the consequent part of the SIRM-\( i \). \( A_i^j \) and \( C_i^j \) are the membership functions of the \( x_i \) and \( \Delta u_i \) in the \( j \)th rule of the SIRM-\( i \), respectively. Further, \( i = 1, 2, \ldots, n \) is the index number of the SIRMs, and \( j = 1, 2, \ldots, m \) is the index number of the rules in the SIRM-\( i \).

If the observation value of the variable \( x_i (i = 1, 2, \ldots, n) \) is given as \( x_i^0 \), the agreement of the antecedent part of the \( j \)th rule in the SIRM-\( i \) simply becomes \( A_i^j(x_i^0) \). Supposing the output universe of discourse is \( U \), the inference result \( \Delta u_i^0 \) of the SIRM-\( i \) can be conducted by

\[
\Delta u_i^0 = \frac{\int_U [\bigoplus_{j=1}^{m_i} (A_i^j(x_i^0) \otimes C_i^j)](\Delta u) \, d(\Delta u)}{\int_U [\bigoplus_{j=1}^{m_i} (A_i^j(x_i^0) \otimes C_i^j)] \, d(\Delta u)}.
\]

(2)

Here, \( \otimes \) means the operator between the agreement and the membership function of the consequent part such as ‘min’ or ‘product’. On the other hand, \( \bigoplus \) means the operator on all the fuzzy rules in the same SIRM, and ‘max’ or ‘sum’ is the typical one. Therefore, the well-known min–max-gravity method [11], product–sum-gravity method [11,13], and simplified inference method [13,14] can be used here.

Usually, each input item is considered to take an unequal role in control performance. Among the input items, some may contribute significantly to control performance, while the contribution of the others is relatively small. Some input items may improve control performance more if their roles are strengthened, while the others may not have a positive influence on control performance if emphasized. Hence, assigning larger weights to such input items that positively contribute to control performance, and assigning at the same time smaller weights to the other input items to restrict their roles, would be in accordance with experts’ experience and would improve total control performance.

In the SIRMs fixed mode [18,19], an importance degree was introduced for each input item to explicitly indicate the different role of each input item. Because of the introduction of the importance degrees, each input item can be directly managed and can realize a different function with each other. As a result, control performance was improved compared with that by the conventional fuzzy inference model.

However, because the importance degrees in the SIRMs fixed model were fixed through the whole control process, the same importance degrees had to be used even at entirely different situations like the rise stage and the settling stage of a first-order plant. Although this led to the simple structure of the fuzzy controller, further improvement in control performance became difficult. For instance, take the first-order lag plant into consideration. Suppose that the output error and its change are selected as the input items. As well known, the rise time and overshoot are two main indexes to evaluate control performance. And the rise time is contradictory to overshoot. If the rise time is to be shortened, then the value of the importance degree of the output error should be increased. However, the big value will cause large overshoot because of the fixed importance degree. On the other hand, if the importance degree takes a large value at the beginning of the control, and gets smaller gradually when the plant output approaches the desired value, then short rise time and small overshoot can both be achieved. This means that the importance degrees should be dynamically adjusted according to control situations in order to obtain better performance.

Here, a dynamic importance degree is introduced for each input item. Given a base value \( w_i^0 \), a dynamic variable \( \Delta w_i \) and a breadth \( B_i \) for input item \( i (i = 1, 2, \ldots, n) \), then the dynamic importance degree \( w_i^D \) of the input item \( i \) is defined by

\[
w_i^D = w_i^0 + B_i \Delta w_i^0,
\]

as the sum of the base value \( w_i^0 \) and the dynamic value \( B_i \Delta w_i^0 \) where \( \Delta w_i^0 \) is the actual output value of the dynamic variable \( \Delta w_i \). The base value insures the basic function of the input item through a control
process. The dynamic variable, varying in \([0.0, 1.0]\), reflects control situations and should be dynamically tuned in real time. The breadth then sets an upper limit to the range within which the dynamic value varies. Therefore, by tuning the dynamic variable according to control situations, the dynamic value will adjust the corresponding dynamic importance degree between \([w_i^0, w_i^0 + B_i]\).

Based on the SIRMs and dynamic importance degrees, the SIRMs dynamically connected fuzzy inference model is expressed as follows. For each input item, a dynamic importance degree is assigned and a SIRM is constructed. After the value of every dynamic importance degree is determined in real-time style according to control situations, the model output \(\Delta u^0\) is obtained by

\[
\Delta u^0 = \sum_{i=1}^{n} w_i^D \Delta u_i^0, \tag{4}
\]

by summarizing the products of the dynamic importance degree and the fuzzy inference result of each SIRM for all the input items.

Since the importance degrees are related linearly to the model output, those input items with larger importance degrees will contribute more significantly to the system, and those with smaller importance degrees will have less influence on the system. Moreover, since the relation among the input items defined by the importance degrees is linearly mapped to the model output, it is possible to achieve specified control purpose by adjusting intuitively the importance degrees. This resembles the setup of the coefficients in PID control; however, the dynamic importance degrees usually are nonlinear function of the corresponding input items.

Because the consequent variable of each SIRM corresponds to the same output item and the product of the importance degree and the fuzzy inference result of each SIRM is a part of the model output, the importance degree can be regarded as a part of the scaling factor of the output item. Therefore, dynamically tuning the importance degrees means adjusting the scaling factor of the output item according to control situations. Although several methods \([7,8,16]\) of tuning the scaling factors of fuzzy controllers were presented, each of the methods was to repeat modifying the scaling factors based on a global performance criterion after a certain length of time of control until almost optimal result was obtained. The scaling factors during tuning were different from those after tuning; however, they were in fact fixed through each control trial. On the contrary, since each of the importance degrees in the proposed model is supposed to have different values, the variable in the consequent part of each SIRM essentially has a different scaling factor although it corresponds to the same output item. In other words, the output item actually holds a different scaling factor in different SIRM. Also, because the importance degrees are adjusted according to control situations, the scaling factor of each SIRM changes automatically with control situations. Moreover, as to be shown in Sections 3 and 5, because tuning a dynamic importance degree only needs the local information of current state, veritable real-time tuning can be easily realized.

If the breadths of the importance degrees are all set up to zero, the proposed model is essentially equivalent to the SIRMs fixed model. Therefore, the SIRMs fixed model can be considered as a special case of the model proposed here.

### 3. Controller building in constant value control

As well known, to control constant value control systems by fuzzy inference model, the output error, the first-order change in the output error, the second-order change in the output error, etc., are usually used as the input items. Suppose that the desired value and the actual plant output value are, respectively, \(x_r\) and \(y(k)\) at sampling time \(k\). (To be exact, the sampling time should be written as \(k \Delta T\) if the sampling period is \(\Delta T\). But in the following it is abbreviated as \(k\) just for simplicity.) Then the output error \(x_1(k)\), its first-order change \(x_2(k)\), its second-order change \(x_3(k)\) can be expressed by

\[
x_1(k) = x_r - y(k), \tag{5}
\]

\[
x_2(k) = x_1(k) - x_1(k - 1), \tag{6}
\]

\[
x_3(k) = x_2(k) - x_2(k - 1). \tag{7}
\]

Here, \(x_i(k)\) \((i = 1, 2, \ldots, n)\) are chosen as the input items and the change \(\Delta u(k)\) in the manipulated variable as the output item in order to show in detail the
controller building method of the SIRM s dynamically connected fuzzy inference model for constant value control systems. For the other different control systems, the proposed model can be extended similarly.

Constant value control, as the name means, is to reduce the output error, its first-order change and so on rapidly to zero, and make the actual plant output equal to the desired value. When the output error and the like are not zero, the control action that converges these items into zero is needed. Furthermore, if the value of the output error, etc., is big, then large control action is very significant in order to improve the control performance.

It is easily understood from Eq. (5) that if the value of current output error is positive, then the current plant output value is still smaller than the desire value. According to Eqs. (6) and (7), if the value of current first- or second-order error change is positive, then the current output error or first-order error change is larger than the previous one. In the above cases, increasing the value of the control input, i.e., the manipulated variable, is effective to reduce the value of the output error or the error changes.

On the other hand, if the value of current output error is negative, then the current plant output value already exceeds the desired value. If the value of current first- or second-order error change is negative, the current output error or first-order error change is smaller than the previous one. In these situations, the value of the control input should be reduced so that the value of the output error or the error changes gets small.

Therefore, in order to converge the actual plant output value to the desired value and keep the values of all the input items to zero, the membership functions of the variable in the consequent part of each SIRM should be in positive relation with those of the corresponding input item. Based on the above analysis, the SIRM of each input item can be set up as Table 1. Here, the membership functions NB, ZO, PB of the variable in the antecedent part are defined in Fig. 1. Because the simplified inference method is adopted for simplicity, the membership functions of the variable in the consequent part are selected as real numbers in [−1.0, 1.0]. The summation of the products of the fuzzy inference result of the consequent variable \( \Delta u_t(k) \) with the dynamic importance degree for all the SIRM s gives the change \( \Delta u_t(k) \) of the manipulated variable at sampling time \( k \).

<table>
<thead>
<tr>
<th>Antecedent variable</th>
<th>Consequent variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>( x_t(k) )</td>
<td>( \Delta u_t(k) )</td>
</tr>
<tr>
<td>NB</td>
<td>−1.0</td>
</tr>
<tr>
<td>ZO</td>
<td>0.0</td>
</tr>
<tr>
<td>PB</td>
<td>1.0</td>
</tr>
</tbody>
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Fig. 1. Membership functions of the SIRM antecedent variable.

Apparently, positive value is chosen as the membership function of the consequent variable for the positive membership function of the antecedent variable, while negative value is given as the membership function of the consequent variable for the negative membership function of the antecedent variable. When the observation value of the antecedent variable is positive, positive value will be obtained as the fuzzy inference result of the SIRM, causing the manipulated variable to increase. As a result, the actual plant output tends to approach the desired value, or the output error or the error change tends to become small. If the observation value of the antecedent variable is negative, then the output of the SIRM also becomes negative and the manipulated variable is decreased. Consequently, the plant output will be put back to the desired value and the error changes will be restrained.

The membership functions of the antecedent variable or the consequent variable are not necessary to be distributed linearly on the universe of discourse. In some complicated cases, asymmetric setting of the membership functions may be essential. However, the setting here is enough to deal with the usual first- and second-order lag plants to be shown in Section 4.

Since the output of the SIRM s dynamically connected fuzzy inference model, i.e., the change in the manipulated variable, is linear with each importance
Table 2
Rules for each dynamic variable in constant value control

<table>
<thead>
<tr>
<th>Antecedent variable</th>
<th>Consequent variable</th>
</tr>
</thead>
<tbody>
<tr>
<td>$</td>
<td>x_1(k)</td>
</tr>
<tr>
<td>DS</td>
<td>0.0</td>
</tr>
<tr>
<td>DM</td>
<td>0.5</td>
</tr>
<tr>
<td>DB</td>
<td>1.0</td>
</tr>
</tbody>
</table>

Fig. 2. Membership functions for the dynamic variables.

degree, large importance degree will result in large change in the value of the manipulated variable under the same condition. Therefore, from the viewpoint of control, when the absolute value of an input item is large, its corresponding importance degree should be enlarged so that the value of the input item will become small as soon as possible. But if the absolute value of an input item is small, large importance degree will cause the control object to become unstable. In this case, the importance degree should be set up to rather small value in order to maintain the control stability.

This fact indicates that for constant value control systems, it is sufficient to use the magnitude of the current value of each input item to determine its corresponding dynamic importance degree. Based on this fact, fuzzy rules for the dynamic variable of each dynamic importance degree can be simply established as in Table 2. Here, the absolute of the current value of the corresponding input item is taken as its antecedent variable and the dynamic variable $\Delta w_i(k)$ as its consequent variable. Furthermore, the membership functions DS, DM, DB of the antecedent variable are defined in Fig. 2.

Evidently, if the absolute value of an input item is large, the inference result of the corresponding dynamic variable will also increase, causing the importance degree of the input item to increase. As a consequence, the input item comes to hold stronger influence on the control system, and the absolute value of the input item tends to decrease rapidly. If the absolute value of an input item is small, the corresponding dynamic variable will only take small value and the importance degree of the input item will remain almost at its base value. In this case, the input item has weaker influence on the control system if its base value is small.

Since only the local information, i.e., the current value of the corresponding input item is necessary in the antecedent part of the fuzzy rules for each dynamic variable, each dynamic importance degree can be easily tuned in real time. Moreover, for constant value control systems, different input items have completely the same rule setting of the SIRM, and the dynamic variables of different input items can be inferred based on the same setting of the fuzzy rules. Therefore, the SIRMs dynamically connected fuzzy inference model has a simple structure and can be realized easily.

4. Control simulations of constant value systems

To verify the effectiveness of the proposed model, here the model is applied to typical first- and second-order lag plants. In the simulations below, the sampling period is fixed to 0.1 s.

4.1. First-order lag plants

The transfer function of the first-order lag plants can be expressed by

$$G(s) = \frac{Ke^{-Ls}}{1 + Ts},$$

where parameters $K$, $L$, $T$ are the process gain, dead time, time constant, respectively. For the desired value 1.0, three plants (plant (1): $K = 1.0$, $L = 1.0$, $T = 10.0$; plant (2): $K = 1.0$, $L = 1.0$, $T = 1.0$; plant (3): $K = 1.0$, $L = 1.0$, $T = 0.1$) [14,18] are selected for control simulation.

To construct the fuzzy controller for the first-order lag plants, the output error $x_1(k)$ and its first-order
change $x_2(k)$ at sampling time $k$ is chosen as the input item, and the change $\Delta u(k)$ in the manipulated variable as the output item. Then, the SIRMs fixed model is composed of two SIRMs (Table 1). The $x_1(k)$ and $x_2(k)$ are separately used as the single variable in the antecedent part of each SIRM, and the change in the manipulated variable is obtained by summarizing the products of the fixed importance degrees and the fuzzy inference results $\Delta u_1^0(k)$, $\Delta u_2^0(k)$ of the two SIRMs. On the other hand, the model proposed here also needs two fuzzy rule modules (Table 2) for the dynamic variables, in addition to the two SIRMs.

The simulation results are indicated in Figs. 3–5 where (a) is the result by the SIRMs fixed model, and (b) is the result by the proposed model using the same set of the scaling factors as that of (a). $S(1.0000, 0.0100, 0.1000)$ shows the set of the scaling factors of the two input items and the output item. In the three simulations, the same set of the scaling factors is adopted. In the SIRMs fixed model, $W(0.9360, 0.5000)$ is a set of the fixed importance degrees of the two input items. In the newly proposed model, $W(0.2000, 0.2000)$ and $B(1.1230, 0.4000)$ are a set of the base values and a set of the breadths of the dynamic importance degrees, respectively. All the parameters are selected by trial and error here. Tuning optimally the parameters by the genetic algorithm [3] is a future subject. Further, the dotted line illustrates the desired value, and the upper and lower curves correspond to the controlled variable (the plant output) and the manipulated variable (the control input).

As can be seen from Figs. 3–5, by using the newly proposed model, the reaching time (from starting to reaching the desired value) is shortened, respectively by about 20%, 15% and 20% compared with that of the SIRMs fixed model. As another performance index, the ITAE of the plant (1) decreases from 152.143 by the SIRMs fixed model to 114.251 by the proposed model. The ITAE of the plant (2) is reduced from 66.073 by the SIRMs fixed model to 52.338 by the proposed model, and the ITAE of the plant (3) becomes from 24.661 by the SIRMs fixed model to 18.226 by the proposed model. This is because the dynamic importance degrees can be set up to larger values at the control beginning so as to make the plant output rise quickly, and get smaller when the plant output approaches to the desired value so that no overshoot or vibration occurs.

![Fig. 3. Control results of the first-order lag plant (1). (a) By the SIRMs fixed model; (b) By the newly proposed model.](image)

4.2. Second-order lag plants

The transfer function of the second-order lag plants discussed here is written as

$$G(s) = \frac{A}{s^2 + Bs + C}e^{-Ls},$$ (9)

where $A, B, C$ are coefficients and $L$ is the dead time. Given the desired value 60.0, three plants (plant (1): $A = 1.228$, $B = 0.6380$, $C = 0.0340$, $L = 0.0$; plant (2): $A = 19.54$, $B = 0.4000$, $C = 0.5400$, $L = 0.0$; plant (3): $A = 0.231$, $B = 0.0994$, $C = 0.0064$, $L = 1.0$) [18] are selected as the control objects.

To realize the fuzzy control of the second-order lag plants, the output error $x_1(k)$, its first-order change $x_2(k)$, its second-order change $x_3(k)$ are adopted as the input items, and the change $\Delta u(k)$ in the manipulated variable as the output item. Then, the SIRMs fixed model consists of three SIRMs shown in Table 1. The proposed model includes the same three SIRMs.
three more fuzzy rule modules shown in Table 2 for tuning the three dynamic importance degrees.

The simulation results obtained by the SIRMs fixed model and the newly proposed model are depicted in Figs. 6–8. Here also, the same set of the scaling factors is used in the three simulations. By using the newly proposed model, the reaching time of the plant (1) and the plant (2) both is shortened by about 20%, and the reaching time of the plant (3) is decreased by more than 30%. The ITAEs of the three plants are reduced separately from 5796.182, 1240.981, and 24916.753 by the SIRMs fixed model to 3290.642, 765.184, and 13170.867 by the proposed model. Therefore, the proposed model apparently can improve largely the control performance.

4.3. Discussions

As defined in Eqs. (5)–(7), since the output error is the difference between the desired value and the actual plant output value, it mainly works at the rise stage, or at such an interval that vibration occurs, or at the settling stage where the steady-state error is still left. If the importance degree of the output error is large, then the plant output tends to rise rapidly or get strong against disturbance.

For constant value control systems, the first-order change in the output error means essentially the value change in the plant output. When the importance degree of the first-order error change is large, the value of the plant output is to be restrained to the same one. As a result, vibration can be prevented although the plant may rise slowly.

Furthermore, the second-order change in the output error actually equals to the first-order change in the first-order error change. If the importance degree of the second-order error change is given with a big value, then the first-order error change tends to vary little. In this case, the plant output has a tendency of rising straight to the desired value or keeps constant since
the first-order error change corresponds to the gradient of the plant output value.

In Figs. 3–8, although the base value of each input item in the proposed model is smaller than or near the corresponding fixed importance degree used in the SIRMs fixed model, the summation of the base value and the breadth of each input item is bigger than the fixed one. Then, at the beginning of control, the dynamic importance degree of the output error takes large value, causing the manipulated variable to increase. Consequently, the plant output rises quickly. When the plant output approaches to the desired value, the importance degree of the output error becomes small and the importance degree of the first-order error change or the second-order error change becomes relatively significant. Therefore, the plant output can reach the desired value quickly and smoothly, and no overshoot or steady-state error is caused.

Note that the same desired value is given to the three first-order lag plants, or to the three second-order lag plants. And the set of scaling factors is also set up identically in the three first-order lag plants, or the three second-order lag plants. For an input item, the scaling factor is used to normalize it from its real specification to $[-1.0, +1.0]$. For an output item, the scaling factor is to put it back from $[-1.0, +1.0]$ to its real specification. On the other hand, the plant output is linear with the control input according to the difference equation of time-invariant systems. If different desired value is given, changing the control input in the same ratio would produce a similar control result. Therefore, if any other desired value is requested in the above plants, it is sufficient just to zoom all the scaling factors in the same ratio, which equals to the new desired value divided by the desired value given in Figs. 3–8, without changing the parameters of the dynamic importance degrees. In this way, the output of the SIRMs dynamically connected fuzzy inference model will be independent of the desired value.
5. Stabilization control of inverted pendulum system

To further show the effectiveness of the proposed model, stabilization control of an inverted pendulum system including the position control of the cart is considered here. The inverted pendulum system consists of a straight-line rail, a cart, and a pendulum. Suppose that no friction exists in the system. The pendulum angle is positive if the pendulum is clockwise from the upright position. The cart position is positive if it locates at the right side of the rail origin. The driving force is positive if it moves the cart toward the right direction. Stabilization control is to balance the pendulum upright and put the cart to the origin of the rail by moving the cart right and left.

5.1. Stabilization fuzzy controller

As well known from experience, if the position control of the cart takes priority over the angular control of the pendulum, then the angular control of the pendulum will fail. Therefore, the angular control of the pendulum should have priority over the position control of the cart when the pendulum does not stand up yet, and the position control of the cart should be done after the pendulum is almost balanced upright. Because the dynamic importance degree shows the influence strength on system performance and hence can signify the priority orders, the stabilization fuzzy controller can be constructed as follows by using the SIRMs and the dynamic importance degrees. Here, the four state variables (pendulum angle, pendulum angular velocity, cart position, and cart velocity) after normalization by their scaling factors are chosen in this order as the input items \( x_i \) \( (i = 1, 2, 3, 4) \). The driving force after normalization by its scaling factor is selected as the output item \( f_i \).

For each of the four input items, the SIRM can be determined by Table 3, where the membership functions NB, ZO, PB are also defined in Fig. 1. Consequent variable \( f_i \) is an intermediate variable corresponding to the output item \( f \) of the fuzzy controller. By this setting, when the angle and the angular velocity of the pendulum are positive, positive driving force is generated so that the cart moves toward the right direction. As a result, the pendulum will rotate counterclockwise toward the upright position, and the angle and the angular velocity will tend to decrease toward zero. If the position and the velocity of the cart are positive, positive driving force moves the cart further toward the right direction, causing the pendulum down to the negative direction deliberately. Because the angle and the angular velocity of the pendulum become negative then, negative driving force is outputted to move the cart toward the rail origin. As a result, the pendulum is balanced upright and the cart is returned to the rail origin.

Table 3

<table>
<thead>
<tr>
<th>Antecedent variable ( x_i ) ( (i = 1, 2, 3, 4) )</th>
<th>Consequent variable ( f_i ) ( (i = 1, 2, 3, 4) )</th>
</tr>
</thead>
<tbody>
<tr>
<td>NB</td>
<td>−1.0</td>
</tr>
<tr>
<td>ZO</td>
<td>0.0</td>
</tr>
<tr>
<td>PB</td>
<td>1.0</td>
</tr>
</tbody>
</table>
Table 4
Rules for the two dynamic variables of the pendulum

<table>
<thead>
<tr>
<th>Antecedent variable</th>
<th>Consequent variable $\Delta w_i$ ($i = 1, 2$)</th>
</tr>
</thead>
<tbody>
<tr>
<td>DS</td>
<td>0.0</td>
</tr>
<tr>
<td>DM</td>
<td>0.5</td>
</tr>
<tr>
<td>DB</td>
<td>1.0</td>
</tr>
</tbody>
</table>

The pendulum system [4,10] is selected. The pendulum mass is 0.1 kg, the pendulum full length is 1.0 m, and the cart mass is 1.0 kg. The angle of the pendulum is limited to $[-30.0^\circ, +30.0^\circ]$, and the moving range of the cart is limited to $[-2.4 \text{ m}, +2.4 \text{ m}]$. The scaling factors of the four input items are fixed to $30.0^\circ$, $100.0^\circ$/s, 2.4 m, and 1.0 m/s, respectively. The scaling factor of the output item is set up to 10 times the total mass of the pendulum and the cart.

After trial and error, the base values of the four input items are separately set to 2.00, 1.50, 0.15, and 0.15, and the breadths of the four input items are separately set to 2.50, 1.00, 0.20, and 0.20. That is, the base value and the breadth of the pendulum angle almost equal to the base value and the breadth of the angular velocity, and the base value and the breadth of the cart position almost equal to the base value and the breadth of the cart velocity. Moreover, the base values and the breadths of the pendulum are about 10 times as large as those of the cart.

Table 4 shows the fuzzy rules for the dynamic variables of the dynamic importance degrees of the angle and the angular velocity of the pendulum. The absolute value of the pendulum angle after normalization is selected as the only antecedent variable. Fig. 2 also indicates the membership functions DS, DM, DB. By this setting, if the absolute value of the pendulum angle is big, the values of the two dynamic variables increase so that the two dynamic importance degrees of the pendulum become large. Consequently, the angular control of the pendulum is stressed, making the pendulum rotate toward the upright position. If the absolute value of the pendulum angle is near zero, the values of the two dynamic variables decrease such that the two importance degrees of the pendulum become small. As a result, the influences of the two input items are weakened and the almost balanced state is kept.

The dynamic variables of the dynamic importance degrees of the position and the velocity of the cart can be inferred by using the fuzzy rules of Table 5 which also takes the absolute value of the pendulum angle as the antecedent variable. By this setting, when the pendulum is far from the upright position, the two importance degrees of the cart are reduced because the values of the two dynamic variables become small. Therefore, the angular control of the pendulum can take priority over the position control of the cart. When the pendulum is almost balanced upright, the values of the two dynamic variables get large such that the two importance degrees of the cart are increased, making the position control of the cart to start.

It is clear that the setting order of the real number membership functions in Table 3 for the cart is just reverse to that of Table 2 for the pendulum. By this setting, the inference results of Tables 2 and 3 become complementary with each other. If the pendulum is not balanced upright yet, the two importance degrees of the pendulum become much bigger than the two importance degrees of the cart because the base values and the breadths of the pendulum are bigger and the two dynamic variables of the pendulum take bigger values. Consequently, the angular control of the pendulum takes priority over the position control of the cart and makes the pendulum rotate toward the upright position. On the contrary, if the pendulum is almost balanced upright, the two importance degrees of the pendulum decrease almost to their base values and the two importance degrees of the cart increase almost to the sums of their base values and breadths. Although either of the base values of the pendulum is still larger than the sum of the base value and the breadth of either dynamic importance degree of the cart, the inference result of the SIRM corresponding to the pendulum angle becomes almost zero. As a result, the contribution of the cart to the output of the fuzzy controller will exceed the contribution of the pendulum so that the position control of the cart is started. In this way, the priority orders of the
pendulum angular control and the cart position control are switched smoothly by adjusting automatically the dynamic importance degrees according to control situations.

5.2. Stabilization control simulations and discussions

Figs. 9 and 10 show control results of the inverted pendulum system by the stabilization fuzzy controller. The sampling period is 0.01 s. At 5.0 s, a disturbance of 15.0° is added to the pendulum angle. The left axis and the right axis indicate separately the pendulum angle and the cart position, respectively.

In Fig. 9, the initial angle of the pendulum is 30.0° and the other initial values are all zeros. Since the initial angle of the pendulum is positively big, the angular control of the pendulum takes priority over the position control of the cart. As a result, the cart is first driven from the rail origin to the right side such that the pendulum rotates counterclockwise toward the upright position. After the pendulum reaches the upright position, it still rotates to the negative direction because of its angular velocity. Then the negative driving force moves the cart back toward the rail origin, causing the pendulum to stand up.

In Fig. 10, the initial position of the cart is set to 2.0 m and the other initial values are all zeros. Because initially the pendulum stands upright and the cart locates at the right side of the rail, the position control of the cart is started from the beginning. Positive driving force moves the cart further away from the rail origin, making the pendulum a little down to the negative direction. Then the angular control of the pendulum takes priority over the position control of the cart and generates negative driving force. As a result, the pendulum is balanced upright and the cart is returned to the rail origin.

Without the disturbance, the pendulum system can be stabilized in about 8.0 s in either of the cases. When the disturbance occurs, the almost balanced state is severely destroyed. Because the dynamic importance degree of the pendulum angle increases when the pendulum angle is disturbed, however, the angular control of the pendulum is executed immediately with priority over the position control of the cart. Consequently, as can be seen from Figs. 9 and 10, the stabilization control of the pendulum system is successfully realized in spite of the disturbance.

As a comparison, Kandadai [4] presented a fuzzy-neuro architecture to automatically generate a fuzzy knowledge base by a pseudo-supervised learning scheme. It took more than 12.0 s, however, to asymptotically stabilize a pendulum system with some offset besides its structural complexity. Kawaji [5] constructed a simple fuzzy controller that imbedded the position control of the cart as a virtual target angle into the angular control of the pendulum, but the controller was difficult to completely stabilize a pendulum system within a short time. Kyung [6] designed a fuzzy controller, whose rule base was
derived from three neural networks. Although the fuzzy controller stabilized a pendulum system in about 8.0 s, it needed 396 rules and a complicated tuning scheme. Sakai [15] applied a nonlinear optimization method to train a stabilization fuzzy controller, which had one rule base for the pendulum and another for the cart. Because the two kinds of the fuzzy rules were not distinguished in composition, however, the controller spent more than 200.0 s on complete stabilization.

Because the pendulum angular control is in conflict with the cart position control, handling equally the four input items results in inefficiency and complexity. The proposed fuzzy controller uses the dynamic importance degrees to definitely discriminate the input items and adjust the priority orders of the pendulum angular control and the cart position control according to control situations. As a result, the proposed fuzzy controller with a simple and intuitively understandable structure can completely stabilize the inverted pendulum system in short time.

6. Conclusions

The SIRMss dynamically connected fuzzy inference model is newly proposed for plural input fuzzy control. The dynamic importance degree is defined as the sum of a base value insuring the role of the input item through a control process, and a dynamic value varying with control situations to adjust the influence of the input item. Because the dynamic importance degrees are allowed to change with control situations, the proposed model is situation variant. The building method of the fuzzy controllers is given for constant value control systems, and control simulations of first- and second-order lag plants are done. The simulation results show that compared with the SIRMss fixed importance degree connected fuzzy inference model, by using the proposed model the reaching time is reduced by more than 15% and the ITAE is decreased by more than 20% without steady-state error and overshoot. Stabilization control of an inverted pendulum system is also executed. By using the SIRMss and the dynamic importance degrees to switch the priority orders of the pendulum angular control and the cart position control, the fuzzy control based on the proposed model is shown to stabilize the inverted pendulum system completely in about 8.0 s.

References

[16] H. Ying, L.C. Sheppard, Tuning parameters of the fuzzy controller based on the golden section search, Proc. 9th

