Maximum-Likelihood Deformation Analysis of Different-Sized Fingerprints *

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Abstract. This paper introduces a probabilistic formulation in terms of Maximum-likelihood estimation to calculate the optimal deformation parameters, such as scale, rotation and translation, between a pair of fingerprints acquired by different image capturers from the same finger. This uncertainty estimation technique allows parameter selection to be performed by choosing parameters that minimize the deformations uncertainty and maximize the global similarity between the pair of fingerprints. In addition, we use a multi-resolution search strategy to calculate the optimal deformation parameters in the space of possible deformation parameters. We apply the method to fingerprint matching in a pension fund management system in China, a fingerprint-based personal identification application system. The performance of the method shows that it is effective in estimating the optimal deformation parameters between a pair of fingerprints.

1 Introduction

Fingerprint-based biometric systems have attracted great interest of researchers to find new algorithms and techniques for fingerprint recognition in the last decade. Great progress has been made in the development of on-line fingerprint sensing techniques [1, 2, 3] and, as a consequence, several small and inexpensive sensing elements have overrun the market. Significant improvements have been achieved on the algorithmic side as well [2, 3]. However, a large number of challenging problems [4,5] still exist. For example, a pension fund management system in China, a fingerprint-based personal identification application system, requires that the matching

* This paper is supported by the National Science Fund for Distinguished Young Scholars of China under Grant No. 60225008, the Special Project of National Grand Fundamental Research 973 Program of China under Grant No. 2002CCA03900, the National High Technology Development Program of China under Grant No. 2002AA234051, the National Natural Science Foundation of China under Grant Nos. 60172057, 69931010, 60071002,30270403,60072007;

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J. Kittler and M.S. Nixon (Eds.): AVBPA 2003, LNCS 2688, pp. 421-428, 2003.

algorithm be more tolerant with deformations and non-linear distortions, which influence the performance of these algorithms in fingerprints acquired by different image capturers.

Although many methods [3,4,5] have been proposed and succeeded in dealing with many similar problems mentioned above, they, to the best of our knowledge, are only designed for identifying a pair of fingerprints acquired by the same image capturer and may be invalid when applied to fingerprint identification for a pair of fingerprints produced by different image capturers. In addition, it is very difficult for many researchers to find an effective and efficient optimization algorithm capable of automatically calculating the optimal deformation parameters between the pair of fingerprints.

Thereby, we introduce in this paper a probabilistic formulation in terms of Maximum-likelihood estimation to automatically model the deformations, such as scale, rotation, and translation, between a pair of fingerprints. This uncertainty estimation technique adopts the strategy of replacement of the local similarity between two fingerprints by their global similarity in estimating their deformation parameters. To search the optimal deformation parameters in the space of possible deformation parameters, we use a multi-resolution search strategy in examining the hierarchical cell of this space of possible deformation parameters, by which this space is divided into cells, and then the algorithm can determine which cells could contain a position satisfying the acceptance criterion. The motivation of our effort is that a good comprehension of the deformation dynamics can be very helpful for designing new robust (deformation tolerant) fingerprint matching algorithms. The performance of our method proves that it is an effective method of dynamically estimating the deformation parameters between a pair of fingerprints.

2 Deformation Analysis

Pressing the finger's tip against the plain surface of an on-line acquisition sensor produces, as the main effect, a 3d to 2d mapping of the finger skin. The user's random placement brings out the deformations, such as rotation and translation, between a pair of fingerprints acquired by the same image capturer from the same finger. There is also scale deformation between a pair of fingerprints acquired by different image capturers. These deformations in fingerprints greatly influence the performance of a fingerprint-matching algorithm.

Many methods [1,8,9] in literature explicitly attempt to model fingerprint deformations for a pair of fingerprints acquired by the same image capturers. However, few are designed for a pair of fingerprints acquired by different image capturers. And many fingerprint-based personal identification application systems, such as the mentioned pension fund management system in China, require that their matching algorithms be tolerant with some factors, such as resolution and size, which these image capturers bring out, and therefore greatly influence the performance of these algorithms. Hereby, to model deformations between a pair of fingerprints acquired by different image capturers for fingerprint matching, we investigate the characteristics of mapping and introduce a probabilistic formulation in terms of maximum-likelihood estimation.

2.1 Fingerprint Minutiae

A fingerprint is the pattern of ridges and valleys on the surface of a finger. The uniqueness of a fingerprint can be determined by the overall pattern of ridges and valleys as well as the local ridge anomalies (a ridge bifurcation or a ridge ending, call minutiae points) which posses the discriminatory information. Our representation for the fingerprint consists of positional, directional, and type information of minutiae. Let $M(F)=\{(x_i, y_i, \alpha_i, \beta_i)\}$, $i=1,2,\ldots,m(F)$ be the minutiae set of fingerprint F containing the position information (x, y), directional information (α), and minutiae type information β (β=0 indicates a ridge ending and β=1 indicates a bifurcation) for $m(F)$ minutiae elements in fingerprint F. Parameter $m(F)$ is the number of minutiae in fingerprint F. For convenience, $M(F, i)$ is used to represent the ith minutia M_i in fingerprint F.

2.2 Affine Transformation

For a pair of fingerprints F and G acquired from the same finger, an ink-on-paper fingerprint and a live-scanned fingerprint, there are deformations, including translation, scale, rotation and so on, between them. Let $M(F)=\{M(F, i)=(x_i,y_i,\alpha_i,\beta_i), 1\leq i\leq m(F)\}$and $M(G)=\{M(G, j)=(x_j, y_j,\alpha_j,\beta_j), 1\leq j\leq m(G)\}$ to be the minutiae sets of fingerprint F and G respectively where m(F) and m(G) are the numbers of minutiae elements in fingerprint F and G respectively. Both $M(F)$ and $M(G)$ can be considered to be sets of discrete points at locations of the occupied pixels in fingerprint F and G.
Considering minutiae sets: $M(F)$ and $M(G)$, we impose an affine transformation model T that relates these two minutiae sets, i.e. T: $M(\mathrm{F})\rightarrow M(G)$. The parameterized affine transformation model T is invoked when it can be safely assumed that spatial variations of pixels in a region can be represented by a low-order polynomial [7]. Five random variables θ ($-90<\theta\leq 90$), t_x, t_y, s_x, and s_y (s_x, s_y >0) in the affine transformation model are used to describe the rotation, scale, translation deformations between fingerprint F and G where t_x and t_y represent horizontal and vertical translation respectively, s_x and s_y correspond to horizontal and vertical scale respectively, θ corresponds to rotation. These random variables can be thought of five functions that map the input fingerprint minutiae set, $M(F)$, into the template fingerprint G. But, the type of $M(F, i)$ keeps invariable. The new minutia $M(F, i)$ can be presented by $(x'_i, y'_i, \alpha'_i, \beta_i) = (s_x\cos\theta+s_y \sin\theta+ t_x, -s_x\sin\theta+s_y\cos\theta+ t_y, \arctan[(s_y\alpha_i)/ s_x], \beta_i)$.

2.3 Constructing the Probability Density Function (PDF)

$M(F)$, the minutiae set of fingerprint F, is mapped into the template fingerprint G using the affine transformation model T. To formulate the problem in terms of maximum-likelihood estimation of the deformations, including scale, translation, and rotation, we must have some set of measurements that are a function of these deformation parameters between fingerprints F and G. Similar to methods based on the Hausdorff distance [7], we use the Hausdorff distance from each minutia in fingerprint F (with respect to deformation parameters specified by θ, t_x, t_y s_x, and s_y) to the closest occupied minutia in fingerprint G as our set of measurements. Let d_i (θ, t_x, t_y s_x, s_y) denote these Hausdorff distances. In general, these distances can be found quickly for any θ, t_x, t_y s_x, and s_y, if we pre-compute the Hausdorff distance transform of fingerprint G by Equality 1 on condition that the distance between (x'_i, y'_i) and (x_j, y_j) is less than ε_l and $|a'_i - a'_j| < \varepsilon_a$, where M_i ($1 \le i \le m(F)$) is the ith minutia in fingerprint F after the minutia set $M(F)$ is mapped into the fingerprint G, and M_j ($1 \le j \le m(G)$) in fingerprint G is the jth minutia and the closest minutia to the mapped minutia M_i. The mentioned condition, a bounding box, can accelerate the rate of searching and reduce errors during matching where ε_l and ε_α are thresholds of positional and directional errors. If the mapped minutia M_i meets the mentioned condition, it is reported as the possible position in fingerprint F.

$$d_i(\theta, s_x, s_y, t_x, t_y) = d_i(M_i, M_j) = \left\| M_i - M_j \right\| \tag{1}$$

To yield a matching criterion, A *probability density function* (PDF), a function $p(d_i$ (θ, t_x, t_y s_x, s_y)) ($1 \le i \le m(F)$), is used to measure the global similarity which consists of local minutia similarity. $p(d_i$ (θ, t_x, t_y s_x, s_y)), the PDF of Hausdorff distance, can be obtained by Equality 2 where $\Delta\beta_i = |\beta_i(F) - \beta_j(G)|$, $\sigma^2 = (\varepsilon^2_l + \varepsilon^2_a)$, and if $x=0$, $f(x)=1$; Otherwise, $f(x)=\lambda$ ($0<\lambda\le 1$). $f(x)$ is used to evaluate the type of a minutiae. If the type of $M(F, i)$ in fingerprint F is different from the type of $M(G, j)$ in fingerprint G, their PDF should be discounted by λ ($0<\lambda\le 1$) .

$$p(d_i(\theta, t_x, t_y, s_x, s_y)) = (f(\Delta\beta_i) \times e^{-d_i(\theta, t_x, t_y, s_x, s_y)/2\sigma^2})/(2\pi\sigma^2) \tag{2}$$

2.4 Maximum-Likelihood Measurement

The joint PDF for the Hausdorff distances, given θ, t_x, t_y s_x, and s_y, can be approximated as the product of each individual PDF by Equality 3 where $p(d_i$ (θ, t_x, t_y s_x, s_y)) is PDF of d_i(θ, t_x, t_y s_x, s_y) evaluated by deformation parameter θ, t_x, t_y s_x, and s_y, if the distance measurements are independent. We have found that this yields accurate results since the correlation among Hausdorff distances falls off quickly as the points become farther apart.

$$p(\ldots, d_i(\theta, t_x, t_y, s_x, s_y), \ldots | (\theta, t_x, t_y, s_x, s_y)) = \prod_{i=1}^{m(F)} p(d_i(\theta, t_x, t_y, s_x, s \tag{3}$$

To find the most likely θ, t_x, t_y s_x, and s_y, we find the deformation parameters that maximize Equality 3. It is often easier to work with the logarithm of Equality 3 since this involves addition, rather than multiplication, and yields a measure that preserves the ordering of the deformations.

$$\begin{aligned}&\ln L(d_1(\theta,t_x,t_y,s_x,s_y),\ldots,d_m(\theta,t_x,t_y,s_x,s_y)|(\theta,t_x,t_y,s_x,s_y))\\&=\sum_{i=1}^{m(F)}\ln(f(\Delta\beta_i)/(2\pi\sigma^2))-\sum_{i=1}^{m(F)}d_i(\theta,t_x,t_y,s_x,s_y)/(2\sigma^2)\\&=a-h(\theta,t_x,t_y,s_x,s_y)\end{aligned} \quad (4)$$

a, the first term in equality 4, is irrelative with deformation parameters θ, t_x, t_y, s_x, and s_y, and therefore $\partial a/\partial\theta=\partial a/\partial t_x=\partial a/\partial t_y=\partial a/\partial s_x=\partial a/\partial s_y=0$. $h(\theta, t_x, t_y\ s_x, s_y)$, the second term in equality 4, is a function of θ, t_x, t_y s_x, and s_y, and directly influences the probability density function when deformation parameters θ, t_x, t_y s_x, and s_y change. Theoretically, we seek deformation parameters θ, t_x, t_y s_x, and s_y that maximize this likelihood function on condition that $\partial h/\partial\theta=\partial h/\partial t_x=\partial h/\partial t_y=\partial h/\partial s_x=\partial h/\partial s_y=0$. Thereby we, theoretically, can calculate the optimal deformation parameters θ, t_x, t_y s_x, and s_y in the five-dimension real space, $S=\{(\theta, t_x, t_y, s_x, s_y)$: $\theta\in(-90, 90)$, $t_x\in(-w, w)$, $t_y\in(-h, h)$, $s_x\in(s_{xs}, s_{xl})$, $s_y\in(s_{ys}, s_{yl})\}$ where w and h are the width and height of fingerprint G respectively, s_{xs}, and s_{xl} correspond to the possible minimal and maximal scale horizontal deformations in fingerprint F, and s_{ys} and s_{yl} correspond to the possible minimal and maximal vertical deformations in fingerprint F.

But it may be difficult to find the optimal deformation parameters θ, t_x, t_y s_x, and s_y which meet Equality 11 and maximize this likelihood function a-$h(\theta, t_x, t_y\ s_x, s_y)$ in the discretized space S, because space S may not include the position that maximizes Equality 4. Hereby, we use a search strategy to obtain the possible optimal deformation parameters in discretized space S.

2.5 Search Strategy

To determine these deformation parameters θ, t_x, t_y s_x, and s_y that maximize the likelihood function developed above, we use a search strategy that generalizes previous methods for matching with the Hausdorff distance[8] in the discretized five-dimension space S. This method divides space S into cells and determines which cells could contain a position satisfying the acceptance criterion. The cells that pass the test are divided into sub-cells, which are examined recursively, while the rest are pruned (Fig.1). If a conservative test is used, this method is guaranteed to find the best location in the discretized search space S. The basic idea of the method is to use a multi-resolution search that examines a hierarchical cell decomposition of the space of possible deformation space.

To find whether some cell C in the five-dimension space S may contain a position meeting the criterion (Equality 4), that is, $f_i(\theta, t_x, t_y\ s_x, s_y)\to 0$ (i=1…5), we begin with examining the pose c at the center of the cell C. A five-dimension bound is computed on $|f_i(\theta, t_x, t_y\ s_x, s_y)|$ and treated as a function defined by: $\Delta C=\{\|f_i(\theta, t_x, t_y\ s_x, s_y)\|, i=1,\ldots,5\}$. If $\Delta C<\varepsilon$ (a very litter five-dimension bounding), the search process (as Figure 1) is quilted. Otherwise, we continue searching a minimal bound over the entire cell C. After we have searched over the entire cell C, we select a sub-cell, which minimizes ΔC in C to the next search process.

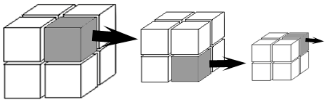

Fig.1. A search strategy used to recursively divides and prunes cells of the space of deformation parameters S.

3 Experimental Results

Our method is used to estimate the possible optimal deformation parameters for the fingerprint-matching algorithm designed specifically for the fingerprint-based pension fund management system in China.

Fingerprint Databases

Two fingerprint databases, an ink-on-paper fingerprint database (Set A) acquired by HP-6350C Scanner and a live-scanned fingerprint database (Set B) acquired by U.are.U-2000 Fingerprint Sensor are produced to evaluate the performance of our method. Each database has 204 fingers and one impression per finger. The size of a fingerprint image in each database is 256×256. But their resolutions are different and the sizes of their available region are obviously different. Fingerprints in Set A and Set B, acquired from fingers of the retirees who are older than 55, are of low quality because the image capturers are of low resolution. Ink-on-paper fingerprints in Set A have the following characteristics: 1) low quality because of being often stained; 2) random placement of a stained finger on a paper. Live-scanned fingerprints in Set B have good placement of a finger in a small rotation direction.

Validation of Maximum-Likelihood in Estimating the Optimal Deformations

For a random pair of fingerprints (such as, 1-a and 1-b, 2-a and 2-b, and 3-a and 3-b in Figure 2) selected from Set A and B respectively, the possible optimal deformation parameters θ, t_x, t_y s_x, and s_y (Table 1) between them are calculated. And minutiae of fingerprints from Set A are aligned by these optimal values and then mapped to these fingerprints of Set B (1-c, and 2-c and 3-c). As the performance above shows, although there are still a few unavoidable false minutiae, which locally influence the accuracy of positioning mapped minutiae, the method have no impact on global estimation of the deformations.

Validation of Maximum-Likelihood in Fingerprint Verification

In matching experiments, Let A_i (i=1…204) and B_j (j=1…204) denote the ith fingerprint in Set A and the jth fingerprint in Set B respectively. If $i=j$, A_i and B_j are considered to be from the same finger. Each fingerprint A_i (i=1…204) in Set A is matched with each fingerprint B_j(j=1…204) in Set B. The number of matches is 204*204=41616. Let R_{ij} denote the matching result between A_i and B_j. If R_{ij} is higher than a certain threshold t, A_i and B_j are considered to be from the same finger. Otherwise, they are considered to be from different fingers. Let n_c ($0 \le n_c \le 204$) denote the number of correct matches, that is, one match between A_i and B_i is considered as a correct match if R_{ii} (i=1…204) $\ge t$. Let n_f ($0 \le n_f \le 41412$) denote the number of false matches, that is, one match between A_i and B_j ($i \neq j$) is considered as a false match if R_{ij} ($1 \leq i,j \leq 204$ and $i \neq j$) $\geq t$. Correct match rate $r_c = (100\% \times n_c)/204$ and false match rate $r_f = (100\% \times n_f)/41412$ are used to describe the performance of our matching algorithm, where r_c is in inverse proportion to r_f according to t. The matching experiment is made on a PC computer with PIII 800 CPU. The average time for matching and enrolling over two fingerprint databases is 2.83 seconds.

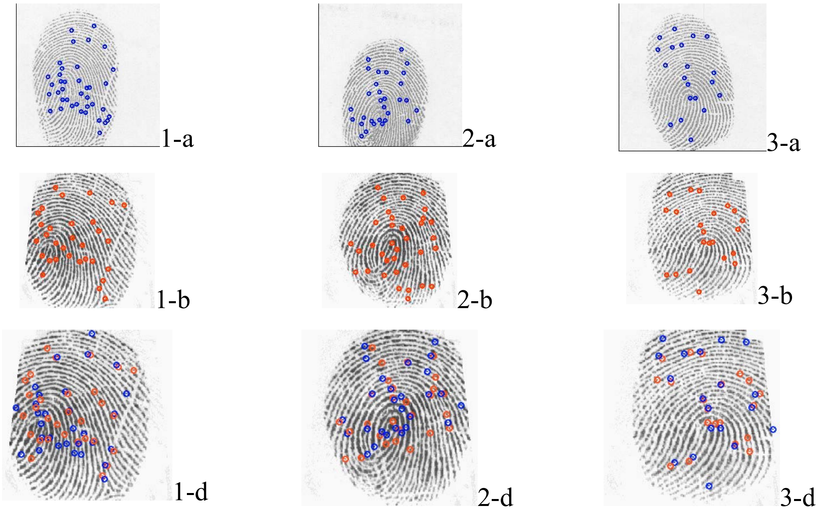

Fig. 2. Performance of our method in estimating the optimal deformations

Table 1. Estimated optimal deformation parameters between three pair of fingerprints

	θ(°C)	s_x	s_y	t_x	t_y
1-a and 1-b	1.90	1.45	1.17	-9.00	-16.03
2-a and 2-b	7.82	1.49	1.15	21.01	-71.00
3-a and 3-b	-3.31	1.49	1.19	35.11	-24.14

Table 2. Performance of our matching algorithm

t	r_c (100%)	r_f(100%)
6.00	*90.45*	10.30
7.00	86.23	8.80
8.00	82.66	7.10
9.00	72.55	5.70
10.00	70.90	2.40
11.00	63.00	1.00
12.00	59.00	0.00

4 Summary and Future Work

This paper introduces a probabilistic formulation in terms of maximum-likelihood estimation to calculate the optimal deformations between a pair of fingerprints. We also use a multi-resolution search strategy to calculate the optimal deformation parameters in the space of possible discretized deformations. But there are non-linear distortions in a finger often produced by individual physical attributes (e.g. force, torque, linear motion, rotation) except the attributes of the image capturer. And it is difficult to use a simple transformation model to simulate these distortions of a fingerprint. R. Cappelli and et al built a plastic distortion model [4] to analyze these non-linear distortions. C.I. Watson and et al proposed a distortion-tolerant filter method [5] for elastic distorted fingerprint matching. These methods provide us with some new ideas and one of our further works is to combine these methods mentioned above and to improve upon our probabilistic formulation. In addition, low-quality fingerprints may bring out many false minutiae that influence the performance of our method. Another future work is to improve upon our method in extracting minutiae from a fingerprint.

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